MT EDUCARE LTD.

SUMMATIVE ASSESSMENT - 1

CBSE - X **2013-14** **Set - C**

Roll No. | | | | | | | |

Code No. 31/1

Series RLH

- Please check that this question paper contains 6 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 34 questions.
- Please write down the serial number of the question before attempting it.

MATHEMATICS

Time allowed **:** 3 hours ***Maximum Marks*** **:** 80

General Instructions:

i) All questions are compulsory.

ii) The question paper consists of 34 questions divided in four sections: A,B,C and D.
Section **A** comprise 10 questions of 1 mark each,
Section **B** comprise 8 questions of 2 marks each,
Section **C** comprise 10 questions of 3 marks each, and
Section **D** comprise 6 questions of 4 marks each.

iii) Question numbers 1 to 10 in Section A are multiple choice questions where you have to select one correct option out of the given four.

iv) There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only of the alternative in all such questions.

v) Use of calculator is not permitted.

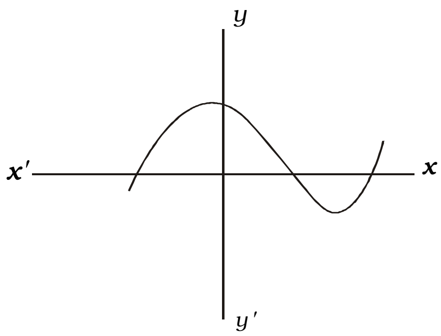
SECTION - A

Question number 1 to 10 carry 1 marks each.

1. For any two positive integers a and b, there exist unique integers q and r such that a = bq + r, $o \leq r < b$, if b = 4, then which is not the value of r?

(A) 1 (B) 2 (C) 3 (D) 4

2. The Graph of y = p (x) given below,
The number of zeroes of p (x) is:



(A) 0 (B) 2 (C) 4 (D) 3

3. The system of equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ has unique solution, if :

(A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (B) $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ (C) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (D) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

4. Cos 1°. cos 2°.cos 3°...... cos 90° is equal to :

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) -1

5. The class mark of the class 29.5 - 30.5 is:

(A) 30 (B) 30.5 (C) 31.5 (D) 31

6. In sin θ = cos θ, then value of θ is :

(A) 0° (B) 45° (C) 30° (D) 90°

7. Given that $\cos\theta = \frac{m}{n}$, then tan θ is equal to :

(A) $\frac{n}{\sqrt{n^2 - m^2}}$ (B) $\frac{\sqrt{n^2 - m^2}}{m}$ (C) $\frac{\sqrt{n^2 - m^2}}{n}$ (D) $\frac{n}{m}$

8. The mean of 6 numbers is 16 with the removal of a number the mean of reamaining numbers is 17. The number removed is :

(A) 2 (B) 22 (C) 11 (D) 6

9. H.C.F. of two consecutive even number is:

(A) 0 (B) 1 (C) 4 (D) 2

10. If α and β are the zeroes of the polynomial $4x^2 + 3x + 7$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to:

(A) $\frac{7}{3}$ (B) $-\frac{7}{3}$ (C) $\frac{3}{7}$ (D) $-\frac{3}{7}$

SECTION - B

Question numbers 11 to 18 carry 2 marks each.

11. Show that any positive integer is of the form 3q,3q + 1 or 3q + 2, where q is some integer.

12. It is being given that 1 is one of the zeroes of the polynomial $7x - x^3 - 6$. Find its other zeroes.

Or

Is the system of linear equations $2x + 3y - 9 = 0$ and $4x + 6y - 18 = 0$ consistent? Justify your answer.

13. Find the mode of the following distribution of marks obtained by 50 students :

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	4	8	10	20	8

14. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

15. Prove that : $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A} = 2 \sec A$.

16. Prove that $\frac{3\sqrt{2}}{5}$ is irrational.

17. Find the value of the expression $\frac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°}$.

18. If tan (A + B) = $\sqrt{3}$ and tan (A - B) = $\frac{1}{\sqrt{3}}$, 0° < A + B $\leq$ 90°; A > B, find A and B.

SECTION - C

Question numbers 19 to 28 carry 3 marks each.

19. Prove that one and only one out of n, $n + 2$ or $n + 4$ is divisible by 3, where n is any positive integer.

20. If $(x + a)$ is a factor of two polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that :

$$a = \frac{n - q}{m - p}$$

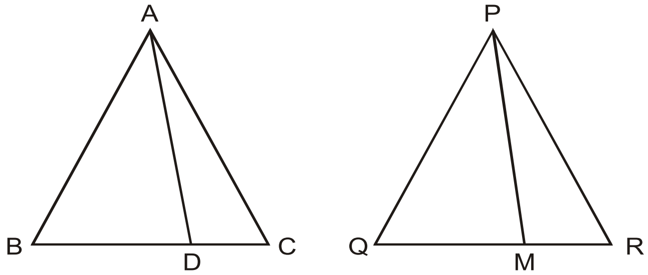
21. Find the zeroes of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ and verify the relation between the zeroes and coefficients of the polynominal.

OR

Solve the following system of linear equations by cross-multiplication method:

$$2(ax - by) + (a + 4b) = 0$$
$$2(bx - ay) + (b + 4a) = 0.$$

22. In fig. (a) and (b), sides AB, BC and median AD of ΔABC are respectively propotional to sides PQ, QR and median PM of Δ PQR. Prove that ΔABC ~ Δ PQR.



(a) (b)

23. The distribution below gives the weight of 30 students of a class. Find the median weight of students.

Weight (in Kg.)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75
No. of Students	2	3	8	6	6	3	2

24. Without using trigonometric tables evaluate :

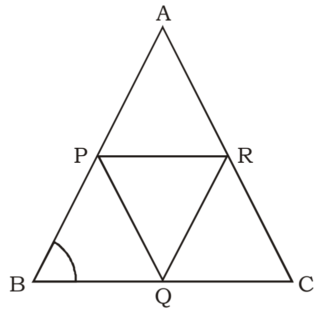
$$2\left[\frac{\cos 58^\circ}{\sin 32^\circ}\right]-\sqrt{3}\left[\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right]$$

OR

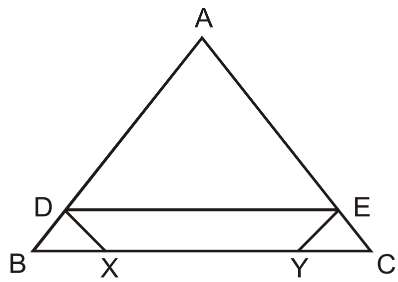
Prove that : $\cos\theta \sin\theta - \frac{\sin\theta \cos(90^\circ-\theta)\cos\theta}{\sec(90^\circ-\theta)} - \frac{\cos\theta \sin(90^\circ-\theta)\sin\theta}{\operatorname{cosec}(90^\circ-\theta)} = 0.$

25. In figure, P, Q and R are respectively the mid-points of sides AB, BC and CA of ΔABC.

Show that ar (PBQR) = $\frac{1}{2}$ ar (ΔABC).



26. In given fig. ΔABC, X and Y are two points lying on the side BC such that BX = CY. If $DX \parallel AC$ and $YE \parallel AB$, then prove that $DE \parallel BC$.



27. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:
$t^2 - 3, \; 2t^4 + 3t^3 - 2t^2 - 9t - 12$

OR

A boat goes 16 km upstream and 24 km downstream in the same time. Find the speed of the boat upstream and downstream.

28. 100 surnames were randomly picked up from a local telephone directory and the frequency distribution of the number of letters in the English alphabets in the surnames was obtained as follows :

Number of letters	1 - 4	4 - 7	7 - 10	10 - 13	13 - 16	16 - 19
Number of surnames	6	30	40	16	4	4

Determine the median number of letters in the surnames.

SECTION - D

Question numbers 29 to 34 carry 4 marks each.

29. Obtain all other zeroes of $2x^4 - 6x^3 + 3x^2 + 3x - 2$, if two of it's zeroes are $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$.

30. In trapezium ABCD, AB || DC, DC = 2AB. EF || AB where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G. Prove that 7EF = 11 AB.

31. Prove that : $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A}$

Or

Show that $n(m^2 - 1) = 2m$, if $\sin\theta + \cos\theta = m$ and $\sec\theta + \operatorname{cosec}\theta = n$.

32. If $m = \tan\theta + \sin\theta$, $n = \tan\theta - \sin\theta$, Show that $m^2 - n^2 = 4\sqrt{mn}$.

33. Find graphically the solution of the equations:

$$x + 2y = 8$$
$$y - x = 4$$

Find the co-ordinates of the points where the two lines meet the y-axis.

OR

Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8 for some integer m.

34. Change the following frequency distribution to less than type distribution and draw its ogive and using it find its median.

Class Interval	100-120	120-140	140-160	160 - 180	180 - 200
Frequency (f)	12	14	8	6	10

The following table gives production yield per hectare of wheat of 100 farms of a village.

All the Best

CBSE X Date :	MT EDUCARE LTD. SUBJECT : **MATHEMATICS** **SUMMATIVE ASSESSMENT - 1** **MODEL ANSWER PAPER**	Set - C Marks : 80 Time : 3 hrs.

Any method mathematically correct should be given full credit of marks.

SECTION - A

1. (D) 4
2. (D) 3
3. (A) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
4. (B) 0
5. (A) 30
6. (B) 45°
7. (B) $\frac{\sqrt{n^2 - m^2}}{m}$
8. (C) 11
9. (D) 2
10. (D) $-\frac{3}{7}$

SECTION - B

11. Euclid's Division Lemma : For any two positive integers a and b, there exists two unique integers q and r such that a = bq + r; $0 \leq r < b$.
If we take b= 3, the possible values of r will be 0, 1 and 2.
Hence either a = 3q
or a = 3q + 1
or a = 3q + 2

12. If 1 is one of the zeroes of $-x^3 + 7x - 6$, then

$x - 1$) $-x^3 + 7x - 6$ ($-x^2 - x + 6$
$-x^3 + x^2$
$+ \quad -$
$- x^2 + 7^x - 6$
$- x^2 + x$
$+ \quad -$
$6x \quad - 6$
$6x \quad - 6$
$- \quad +$
0

Now,
$$- x^2 - x + 6 = -x^2 - 3x + 2x + 6$$
$$= -x(x + 3) + 2(x + 3)$$
$\Rightarrow \quad (x + 3)(2 - x) = 0$
$\Rightarrow \quad x = 2 \text{ or } x = -3$
Hence, other zeroes are -3 and 2.

OR

For the equation $2x + 3y = 9$

$a_1 = 2, b_1 = 3$ and $c_1 = 9$

and for the equation $4x + 6y = 18$

$a^2 = 4, b_2 = 6$ and $c_2 = 18$

Here $\frac{a^1}{a^2} = \frac{2}{4} = \frac{1}{2}, \frac{b^1}{b^2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c^1}{c^2} = \frac{9}{18} = \frac{1}{2}$

$\therefore \quad \frac{a^1}{a^2} = \frac{b^1}{b^2} = \frac{c^1}{c^2}$. Hence system is consistent and dependent.

13.

Marks	**No. of students**
0-10	4
10-20	8
20-30	$10 = f_0$
30-40	$20 = f_1$
40-50	$8 = f_2$

Maximum frequency = 20 (f 1)

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 30 + \frac{20-10}{40-10-8} \times 10$$

$$= 30 + \frac{10}{22} \times 10$$

$$= 30 + \frac{100}{22}$$

$$= 34.55 \text{ (approx)}$$

14. On dividing $3x^4 + 5x^3 - 7x^2 + 2x + 2$by $x^2 + 3x + 1$

$x^2 + 3x + 1) \overline{3x^4 + 5x^3 - 7x^2 + 2x + 2} (3x^2 - 4x + 2$

$3x^4 + 9x^3 + 3x^2$

$- \quad - \quad -$

$- 4x^3 - 10x^2 + 2x + 2$

$- 4x^3 - 12x^2 - 4x$

$+ \quad + \quad +$

$2x^2 + 6x + 2$

$2x^2 + 6x + 2$

$- \quad - \quad -$

0

Reminder is 0 hence $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

15. L.H.S. $= \dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A}$

$= \dfrac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)\cos A}$

$= \dfrac{\cos^2 A + \sin^2 A + 2\sin A + 1}{(1+\sin A)\cos A}$

$= \dfrac{1+2\sin A+1}{(1+\sin A)\cos A}$

$= \dfrac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \dfrac{2}{\cos A}$

= 2 sec A = R.H.S.

16. Let $\dfrac{3\sqrt{2}}{5}$ be a rational number.

$\dfrac{3\sqrt{2}}{5} = \dfrac{a}{b}$, where a and b are co-prime integers and $b \neq 0$

$\Rightarrow \quad \sqrt{2} = \dfrac{5a}{3b}$

Now a, b, 3 and 5 are integers.

$\dfrac{5a}{3b}$ is a rational integers.

$\Rightarrow$ $\sqrt{2}$ is also a rational number which is contradictory

$\therefore$ $\dfrac{3\sqrt{2}}{5}$ is an irrational number.

17. $\dfrac{\cos 30° + \sin 60°}{1 + \cos 60° + \sin 30°} = \dfrac{\sqrt{\dfrac{3}{2}} + \sqrt{\dfrac{3}{2}}}{1 + \left(\dfrac{1}{2}\right) + \left(\dfrac{1}{2}\right)}$

$= \dfrac{2\sqrt{3}/2}{1+1}$

$= \sqrt{3}\ /\ 2$

18. Since, tan (A + B) = $\sqrt{3}$ = tan 60°

$\Rightarrow$ A + B = 60° ....(i)

and tan (A - B) = $\dfrac{1}{\sqrt{3}}$ = tan 30°

$\Rightarrow$ A - B = 30° ... (ii)

On adding (i) and (ii), We get

$$A + B = 60°$$
$$A - B = 30°$$
$$2A = 90°$$
$$A = 45°$$

Putting the value of A in eq. (i), we get

$$A + B = 60°$$
$$45° + B = 60°$$
$$B = 60 - 45°$$
$$= 15°$$

SECTION - C

19. We Know that any positive integer is of the form 3q, 3q + 1 or 3q + 2 for some integer q.

Case I : when $n = 3q$,

$$n = 3q + 0 \Rightarrow n \text{ is divisible by 3}$$
$$n + 2 = 3q + 2 \Rightarrow n + 2 \text{ is not divisible by 3.}$$

and $n + 4 = 3q + 4 = 3(q + 1) + 1 \Rightarrow n + 4$ is not divisible by 3.

Case II : when $n = 3q + 1$,

$$n = 3q + 1 \Rightarrow n \text{ is divisible by 3}$$
$$n + 2 = (3q + 1) + 2 = 3(q + 1) + 0$$

Here remainder is zero, so $(n + 2)$ is divisible by 3

and $n + 4 = (3q + 1) + 4 = 3(q + 1) + 2$

$\Rightarrow$ $(n + 4)$ is not divisible by 3.

Case III : when $n = 3q + 2$.

$$n = 3q + 2 \Rightarrow \text{ is not divisible by 3}$$
$$n + 2 = (3q + 2) + 2 = 3(q + 1) + 1$$

$\Rightarrow$ $n + 2$ is not divisible by 3

and $n + 4 = (3q + 2) + 4 = 3(q + 2) + 0$

Here remainder is zero, so $(n + 4)$ is divisible by 3.

Thus, we conclude that one and only one out of n, $n + 2$ and $n + 4$ is divisible by 3.

20. $(x + a)$ is a factor of $x^2 + px + q$

$$(-a)^2 + p(-a) + q = 0$$
$$a^2 - ap + q = 0$$

$\Rightarrow$

$(x + a)$ is factor of $x^2 + mx + n$

$$(-a)^2 + m(-a) + n = 0$$
$$a^2 - am + n = 0$$

$\Rightarrow$

From (i) and (ii), we get

$$-ap + am + q - n = 0$$
$$a(m - p) = n - q$$
$$a = \frac{n - q}{m - p}$$

21. Zeroes of $4\sqrt{3}\ x^2 + 5x - 2\sqrt{3}$ are given by

$$4\sqrt{3}\ x^2 + 5x - 2\sqrt{3} = 0$$

On comparing with equation $ax^2 + bx + c = 0$

i.e., $a = 4\sqrt{3}$, $b = 5$, $c = -2\sqrt{3}$

$$4\sqrt{3}\ x^2 + 8x - 3x - 2\sqrt{3} = 0$$

$\Rightarrow$ $4x(\sqrt{3}\ x + 2) - \sqrt{3}\ (\sqrt{3}\ x + 2) = 0$

$\Rightarrow$ $(4x - \sqrt{3})(\sqrt{3}\ x + 2) = 0$

$\Rightarrow$ $x = \frac{\sqrt{3}}{4}, \quad -\frac{2}{\sqrt{3}}$

Verification : Sum of Zeroes $= \frac{\sqrt{3}}{4} - \frac{2}{\sqrt{3}}$

$$= \frac{3-8}{4\sqrt{3}} = -\frac{5}{4\sqrt{3}} = -\frac{b}{a}$$

Product of Zeroes $= \frac{\sqrt{3}}{4} \times \frac{(-2)}{\sqrt{3}} = -\frac{2}{4}$

$$= -\frac{1}{2}$$

$$\frac{c}{a} = \frac{-2\sqrt{3}}{4\sqrt{3}} = \frac{-1}{2}$$

$\therefore$ Hence verified.

OR

$$2(ax - by) + a + 4b = 0$$
$$2(bx + ay) + b - 4a = 0$$

From (i), we get

$$2ax - 2by + (a + 4b) = 0$$

From (ii), we get

$$2bx + 2ay + (b - 4a) = 0$$

By cross multiplication method :

$$\frac{x}{(-2b)(b-4a)-(2a)(a+4b)} = \frac{y}{(2b)(a+4b)-(2a)(b-4a)} = \frac{1}{(2a)(2a)-(2b)(2b)}$$

$\Rightarrow$ $\frac{x}{-2b^2-2a^2} = \frac{y}{8b^2+8a^2} = \frac{1}{4a^2+4b^2}$

$\Rightarrow$ $\frac{x}{-2(a^2+b^2)} = \frac{y}{8(a^2+b^2)} = \frac{1}{4(a^2+b^2)}$

$$x = \frac{-2(a^2+b^2)}{4(a^2+b^2)} = -\frac{1}{2}$$

$$y = \frac{8(a^2+b^2)}{4(a^2+b^2)} = 2.$$

Hence $x = -\frac{1}{2}$ and $y = 2$.

22. Given, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$

Since D and M are mid points of BC, QR respectively.

$\therefore \quad \frac{AB}{PQ} = \frac{2BD}{2QM} = \frac{AD}{PM}$

$\Rightarrow \quad \Delta ABD \sim \Delta PQM$

$\Rightarrow \quad \angle B = \angle Q$

In ΔABC and ΔPQR,

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q$

$\therefore \quad \Delta ABC \sim \Delta PQR.$

23.

Weight (in kg.)	**No. of students**	**c.f.**
40 – 45	2	2
45 – 50	3	5
50 – 55	8	*c.f.* = 13
55 – 60 = *l*	*f* = 6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30
	n = 30	

$$\frac{n}{2} = \mathbf{15}$$

$$\text{Medain} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$= 55 + 1.67$$

$$= 56.67$$

24. $2\left[\frac{\cos 58°)}{\sin 32°}\right] - \sqrt{3}\left[\frac{\cos 38° \cos ec\, 52°}{\tan 15° \tan 60° \tan 75°}\right] = 2\left[\frac{\sin(90° - 58°)}{\sin 32°}\right] - \sqrt{3}\left[\frac{\sin(90° - 38°) \cos ec\, 52°}{\tan 15° \times \sqrt{3} \times \cot(90° - 75°)}\right]$

$$= 2\left[\frac{\sin 32°}{\sin 32°}\right] - \sqrt{3}\left[\frac{\sin 52° \times \cos ec\, 52°}{\tan 15° \times \sqrt{3} \times \cot 15°}\right]$$

$$= 2 - \sqrt{3}\left[\frac{\sin 52^\circ \times \dfrac{1}{\sin 52^\circ}}{\tan 15^\circ \times \sqrt{3} \times \dfrac{1}{\tan 15^\circ}}\right]$$

$$= 2 - \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 2 - 1 = 1.$$

OR

L.H.S = $\cos\theta \sin\theta - \dfrac{\sin\theta \cos(90^\circ - \theta)\cos\theta}{\sec(90^\circ - \theta)} - \dfrac{\cos\theta \sin(90^\circ - \theta)\sin\theta}{\operatorname{cosec}(90^\circ - \theta)}$

= $\cos\theta \sin\theta - \dfrac{\sin\theta \sin\theta \cos\theta}{\operatorname{cosec}\theta} - \dfrac{\cos\theta \sin(90^\circ - \theta)\sin\theta}{\operatorname{cosec}(90^\circ - \theta)}$

= $\cos\theta \sin\theta - \dfrac{\sin^2\theta \cos\theta}{1/\sin\theta} - \dfrac{\cos^2\theta \sin\theta}{1/\cos\theta}$

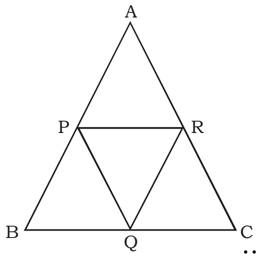
$= \cos\theta \sin\theta - \sin^3\theta \cos\theta - \cos^3\theta \sin\theta$

$= \cos\theta \sin\theta - (\sin\theta \cos\theta)(\sin^2\theta + \cos^2\theta)$

$= \sin\theta \cos\theta - \sin\theta \cos\theta$

= **R.H.S.**

25. Here PR = $\dfrac{1}{2}$ BC, PQ = $\dfrac{1}{2}$ AC, QR = $\dfrac{1}{2}$ AB, [$\because$ P, Q,R are mid-points of AB, BC and CA]



$\Rightarrow$ $\dfrac{PR}{BC} = \dfrac{PQ}{CA} = \dfrac{QR}{AB} = \dfrac{1}{2}$

$\Rightarrow$ $\Delta PQR \sim \Delta CAB.$

$\Rightarrow$ $\dfrac{ar(PQR)}{ar(CAB)} = \dfrac{PQ^2}{CA^2} = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

$\Rightarrow$ ar (PQR) = $\dfrac{1}{4}$ ar (CAB) ......(i)

Now $PR \parallel BC$ and $QR \parallel AB$

$\therefore$ $PR \parallel BQ$ and $QR \parallel PB$

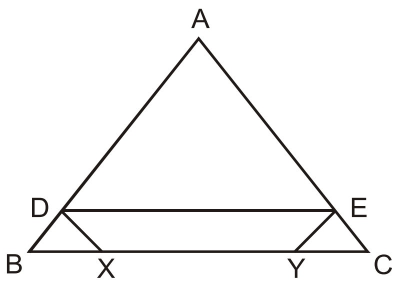
$\Rightarrow$ PBQR is a parallelogram

$\Rightarrow$ ar (PBQR) = 2 × ar (PQR) ......(ii)

From (i) and (ii), We get ar (PBQR) = 2 × $\dfrac{1}{4}$ × ar (CAB) = $\dfrac{1}{2}$ ar (CAB)

Hence Proved

26.



$$BX = CY$$

$$\Rightarrow \quad BX + XY = CY + XY$$

$$\Rightarrow \quad BY = CX$$

Now in ΔABC, $DX \parallel AC \Rightarrow \dfrac{BD}{DA} = \dfrac{BX}{XC}$

$$\Rightarrow \quad \frac{BD}{DA} = \frac{CY}{BY}$$

In ΔABD, $YE \parallel AB \Rightarrow \dfrac{CY}{BY} = \dfrac{CE}{EA}$

From (ii) and (iii), we get $\dfrac{BD}{DA} = \dfrac{CE}{EA}$

or $\dfrac{DA}{BD} = \dfrac{AE}{CE}$

$\Rightarrow$ $DE \parallel AC$ [By converse of BPT]

27.

$$
\begin{array}{r|l}
 & 2t^2 + 3t + 4 \\
t^2 - 3 & 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\
 & 2t^4 \qquad - 6t^2 \\
 & - \qquad\quad + \\ \hline
 & 3t^3 + 4t^2 - 9t \\
 & 3t^3 \qquad - 9t \\
 & - \qquad\quad + \\ \hline
 & 4t^2 - 12 \\
 & 4t^2 - 12 \\
 & - \qquad + \\ \hline
 & 0
\end{array}
$$

Quotient = $\mathbf{2t^2 + 3t + 4}$, Remainder = **0**

Yes, the first polynomial is a factor of the second polynomial since remainder is zero.

OR

Suppose speed of the boat in still water = x km/hr
Speed of the current = y km/hr
Upstream speed = (x - y) km/hr.

By formula,, Time = $\frac{\text{Distance}}{\text{Speed}}$

$$\frac{24}{x+y}+\frac{16}{x-y}=6$$

$$\frac{36}{x+y}+\frac{12}{x-y}=6$$

Putting $\frac{1}{x+y}$ = u and $\frac{1}{x-y}$ = v, we get

$$24u + 16v = 6$$

and $$36u + 12v = 6$$

Solving (iii) and (iv), we get

$$u = \frac{1}{12} \text{ and } v = \frac{1}{4}$$

On putting the values of u and v, we get

$$\frac{1}{x+y}=\frac{1}{12} \text{ and } \frac{1}{x-y}=\frac{1}{4}$$

$$x + y = 12 \text{ and } x - y = 4$$

Speed of the boat downstream = 8 km/hr and upstream = 4 km/hr.

28.

Class	**Frequency**	**c.*f*.**
1- 4	6	6
4 - 7	30	36← c.f
7 - 10	40 ← f	76
10 - 13	16	92
13 - 16	4	96
16 - 19	4	100

Here $\frac{n}{2}=\frac{100}{2}$ = 50 which lies in the class 7 - 10.

∴ Median class is 7 - 10
l = 7, h = 3, f = 40, c.f. = 36

$$\text{Median} = l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h$$

$= \quad 7 + \left(\frac{50-36}{40}\right) \times 3$

$= \quad 7 + \frac{42}{40}$

$= \quad 7 + 1.05$

$\therefore$ **Median = 8.05**

SECTION - D

29. Two zeroes are $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$

One factor is $\left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right)$

i.e.,

$2x^2 - 1)\ 2x^4 - 6x^3 + 3x^2 + 3x - 2\ (x^2 - 3x + 2$

$2x^4 \quad - x^2$

$- \quad +$

$-6x^3 + 4x^2 + 3x$

$-6x^3 \quad + 3x$

$+ \quad -$

$-4x^2 - 2$

$-4x^2 - 2$

$- \quad +$

×

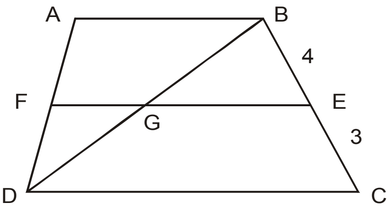
Another factor is $x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = +1$ and $+2$

Hence, other zeroes are 1 and 2.

30.



In trapezium ABCD, $AB \| DC$ and DC = 2AB.

Also $\frac{BE}{EC} = \frac{4}{3}$

In trapezium ABCD, $EF \| AB \| CD$

$$\frac{AF}{FD} = \frac{BE}{BC} = \frac{4}{3}$$

In ΔBGE and ΔBDC,

$$\angle B = \angle B$$
$$\angle BEG = \angle BCD$$
$$\Delta BGE \sim \Delta BDC$$

$\therefore$ $$\frac{EG}{CD} = \frac{BE}{BC}$$

As $\frac{BE}{EC} = \frac{4}{3}$ $\Rightarrow$ $$\frac{EC}{BE} = \frac{3}{4}$$

$\Rightarrow$ $$\frac{EC}{BE} + 1 = \frac{3}{4} + 1$$

$\Rightarrow$ $$\frac{EC + BE}{BE} = \frac{7}{4}$$

$\Rightarrow$ $$\frac{BC}{BE} = \frac{7}{4}$$

$\Rightarrow$ $$\frac{BE}{BC} = \frac{4}{7}$$

$\Rightarrow$ $$\frac{EG}{CD} = \frac{4}{7}$$

$\Rightarrow$ $$EG = \frac{4}{7} CD$$

Similarly $\Delta DGF \sim \Delta DBA$ $\Rightarrow$ $$\frac{DF}{DA} = \frac{FG}{AG}$$

$\Rightarrow$ $$\frac{FG}{AB} = \frac{3}{7}$$

$\Rightarrow$ $$FG = \frac{3}{7} AB$$

$$\left[\begin{array}{l} \because \frac{AF}{FD} = \frac{4}{7} = \frac{BE}{BD} \\ \Rightarrow \frac{EC}{BC} = \frac{3}{7} \end{array}\right]$$

Adding (i) and (ii), we get $$EG + FG = \frac{4}{7} CD + \frac{3}{7} AB$$

$$EF = \frac{4}{7} \times (2AB) + \frac{3}{7} AB = \frac{8}{7} AB + \frac{3}{7} AB = \frac{11}{7} AB$$

$\Rightarrow$ $$7EF = 11 AB.$$

31. L.H.S. $= \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

$= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$

$= \frac{\sin^2 A + 2\sin A \cos A + \cos^2 A + \sin^2 A - 2\cos A + \cos^2 A}{(\sin A - \cos A)(\sin A + \cos A)}$

$= \frac{1+1}{\sin^2 A - \cos^2 A}$

$= \frac{2}{\sin^2 A - \cos^2 A}$

Or

Given, $\sin\theta + \cos\theta = m$

$\sec\theta + \mathrm{cosec}\,\theta = n$

$n = \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$

L.H.S. $= n(m^2 - 1)$

$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)[(\sin\theta + \cos\theta)^2 - 1]$

$= \left(\frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}\right)[\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1]$

$= 2(\sin\theta + \cos\theta)$

= 2*m* = R.H.S.

32. $m^2 - n^2 = (m - n)(m + n)$

$= (2\sin\theta)(2\tan\theta)$

$= 4\sin\theta\tan\theta$

$mn = (\tan\theta + \sin\theta)(\tan\theta - \sin\theta)$

$= \tan^2\theta - \sin^2\theta$

$= \sin^2\theta\frac{(1-\cos^2\theta)}{\cos^2\theta}$

$= \frac{\sin^2\theta\sin^2}{\cos^2\theta} = \tan^2\theta\ \sin^2\theta$

$\sqrt{mn} = \tan\theta\ \sin\theta$

From (i) and (ii), we get $m^2 - n^2 = 4\sqrt{mn}$.

33. $x + 2y = 8 \Rightarrow x = 8 - 2y$

Table of equation (i) is

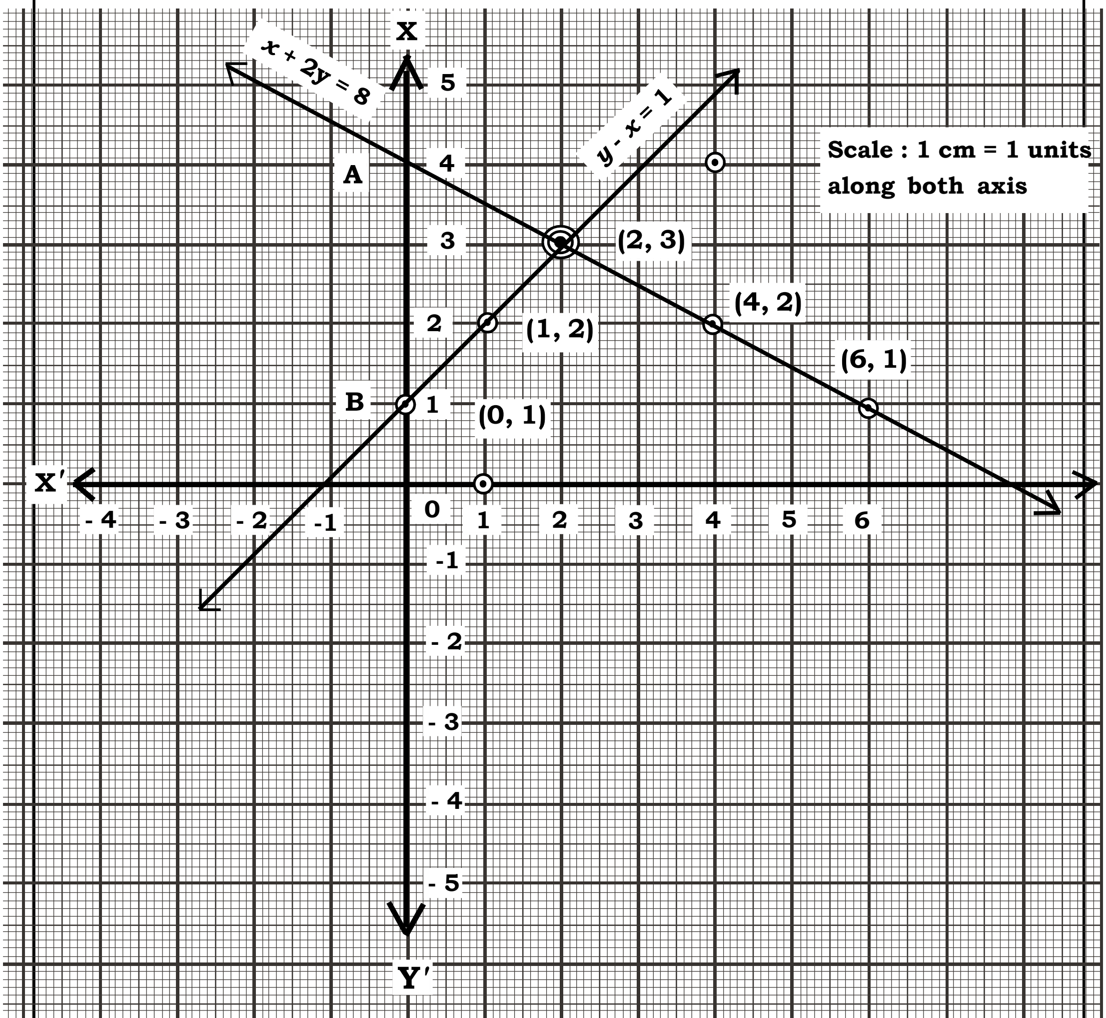
x	6	4	2
y	1	2	3

$y - x = 1 \Rightarrow x = y - 1$

Table of equation (ii) is

x	0	1	2
y	1	2	3

From the tables plotting the graph :



From graph it is clear that the coordinates of the points when the two lines meet y - axis A (0,4) and B (0, 1)
and point of intersection $x = 2$ and $y = 3$.

OR

33. Let x be any positive integer and b = 3

Applying Euclid's Division Algorithm

$\therefore x = 3q + r$ where $0 \leq r < 3$

The possible remainders are 0, 1, 2

$\therefore x = 3q$ or $3q + 1$ or $3q + 2$

i) If $x = 3q \Rightarrow x^3 = (3q)^3 = 27q^3 = 9(3q^3) = \mathbf{9m}$ for some integer m, where $m = 3q^3$

ii) If $x = 3q + 1 \Rightarrow x^3 = (3q+1)^3 = (3q)^3 + 3(3q)^2(1) + 3(3q)(1)^2 + (1)^3$

[$\because$ since $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= 27q^3 + 27q^2 + 9q + 1$

$= 9q(3q^2 + 3q + 1) + 1$

$= \mathbf{9m + 1}$ for some integer m, where $m = q(3q^2 + 3q + 1)$

iii) If $x = 3q + 2 \Rightarrow x^3 = (3q + 2)^3$

$= (3q)^3 + 3(3q)^2(2) + 3(3q)(2)^2 + (2)^3$

[$\because$ since $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= 27q^3 + 54q^2 + 36q + 8$

$= 9q(3q^2 + 6q + 4) + 8$

$= \mathbf{9m + 8}$ for some integer m, where $m = q(3q^2 + 6q + 4)$

cube of any positive integer is either of the form 9*m*, 9*m* + 1 or 9*m* + 8

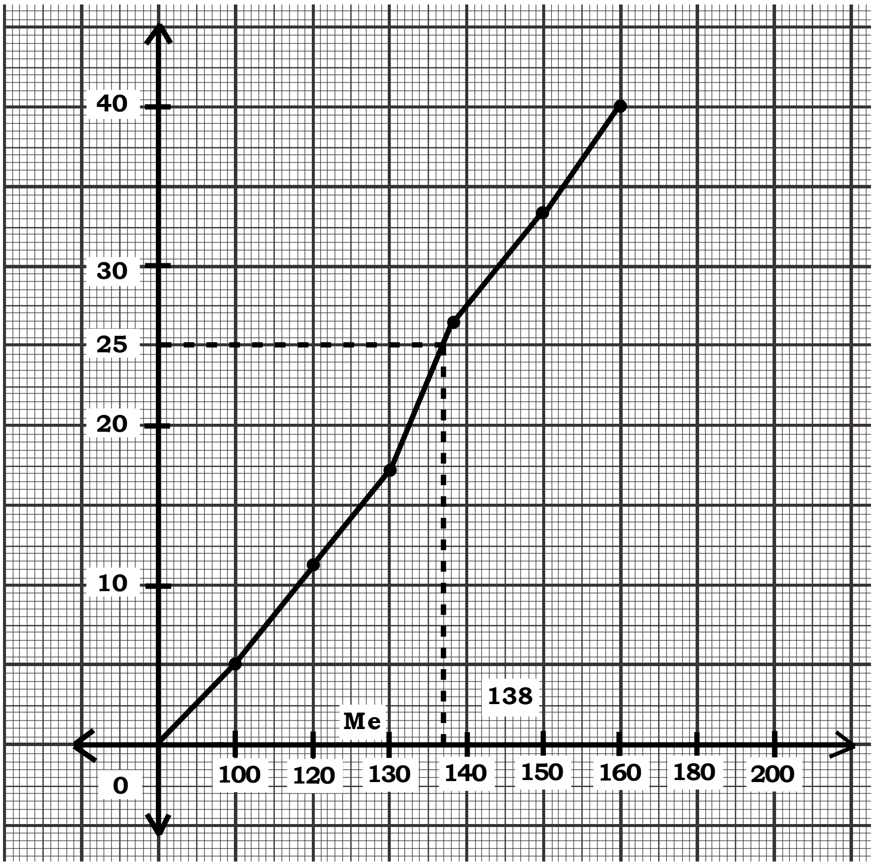
34.

Class Interval	***c.f.***
100 - 120	12
120 - 140	26
140 - 160	34
160 - 180	40
180 - 200	50

Points are (120, 12), (140, 26), (160, 34), (180, 40); (200, 50)

From the graph, $\frac{N}{2} = \frac{34}{2} = 17$

Hence median = 10.2 (approx).



Production yield (in kg/ha.)	Number of farms	Production yield	Cumulative Frequency	Points to be plotted
50 - 55	2	50 or more than 50	100	(50, 100)
55 - 60	8	55 or more than 55	98	(55, 98)
60 - 65	12	60 or more than 60	90	(60, 90)
65 - 70	24	65 or more than 65	78	(65, 78)
70 - 75	38	70 or more than 70	54	(70, 54)
75 - 80	16	75 or more than 75	16	(75, 16)

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