MT EDUCARE LTD.

SUMMATIVE ASSESSMENT - 1

CBSE - X **2013-14** **Set - B**

Roll No. | | | | | | | |

Code No. 31/1

Series RLH

- Please check that this question paper contains 6 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 34 questions.
- Please write down the serial number of the question before attempting it.

MATHEMATICS

Time allowed **:** 3 hours ***Maximum Marks*** **:** 80

General Instructions:

i) All questions are compulsory.

ii) The question paper consists of 34 questions divided in four sections: A,B,C and D.
Section **A** comprise 10 questions of 1 mark each,
Section **B** comprise 8 questions of 2 marks each,
Section **C** comprise 10 questions of 3 marks each, and
Section **D** comprise 6 questions of 4 marks each.

iii) Question numbers 1 to 10 in Section A are multiple choice questions where you have to select one correct option out of the given four.

iv) There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only of the alternative in all such questions.

v) Use of calculator is not permitted.

SECTION - A

Question number 1 to 10 carry 1 marks each.

1. $3.\overline{27}$ is
   (a) an integer (b) a rational number
   (c) a natural number (d) an irrational number

2. If am ≠ bl, then the system of equations
   ax + by = c , lx + my = n
   (a) has a unique solution (b) has no solution
   (c) has infinitely many solution (d) may or may not have a solution

3. The length of the hypotenuse of an isosceles right triangle whose one side is $4\sqrt{2}$ cm is
   (a) 12 cm (b) 8 cm (c) $8\sqrt{2}$ cm (d) $12\sqrt{2}$ cm

4. A quadratic polynomial, the sum of whose zeroes is 0 and one zero is 3, is
   (a) $x^2 - 9$ (b) $x^2 + 9$ (c) $x^2 + 3$ (d) $x^2 - 3$

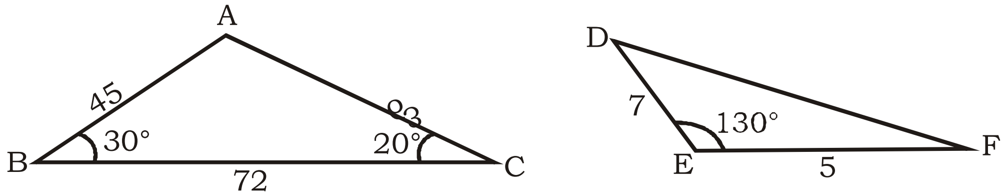
5. The median of a given frequency distribution is found graphically with the help of
   (a) Histogram (b) Frequency polygon
   (c) Ogive (d) Standard deviation

6. If the system of equations 2x + 3y = 5, 4x + ky = 10 has infinitely many solutions, then k =
   (a) 1 (b) ½ (c) 3 (d) 6

7. The HCF of 95 and 152, is
   (a) 57 (b) 1 (c) 19 (d) 38

8. (sec A + tan A) (1 – sin A) =
   (a) sec A (b) sin A (c) cosec A (d) cos A

9. In Fig. 4.242, the measures of D and F are respectively
   (a) 50°, 40° (b) 20° , 30° (c) 40°, 50° (d) 30°, 20°



10. if 8 tan x = 15, then sin x – cos x is equal to

(a) $\frac{8}{17}$ (b) $\frac{17}{7}$ (c) $\frac{1}{17}$ (d) $\frac{7}{17}$

SECTION - B

Question number 11 to 18 carry 2 marks each.

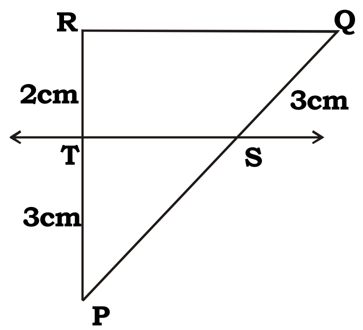
11. Find the mode of following distribution :

Height (in cm)	30-40	40-50	50-60	60-70	70-80
No. of Plants	4	3	6	11	8

12. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

13. Check whether 6^n can end with the digit 0 for any natural number n ?

14. If ST || QR. Find PS.



15. If sin (A+B) = cos (A-B) = $\frac{\sqrt{3}}{2}$ and A,B (A > B) are acute angles, find the values of A and B.

OR

15. If A, B, C are the interior angles of Δ ABC, then prove that cos $\left(\frac{A+B}{2}\right)$ = sin $\frac{C}{2}$.

16. Find the L.M.C and H.C.F. of 15, 18, 45 by the prime factorisation method.

17. Prove that $15 + 17\sqrt{3}$ is an irrational number.

18. Solve the following system of equations by using the method of cross-multiplication :

$2x - y - 3 = 0$

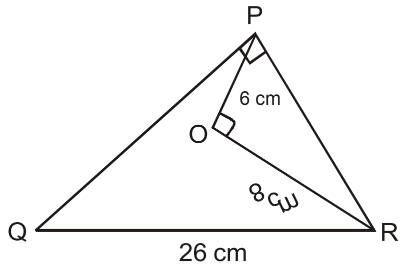
$4x + y - 3 = 0$

SECTION - C

Question numbers 19 to 28 carry 3 marks each.

19. Prove that $\frac{1}{2}-\frac{\sqrt{5}}{3}$ is irrational.

20. Calculate the area of ΔPQR, where OP = 6 cm, OR = 8 cm and QR = 26 cm.
$\angle QPR = \angle POR = 90°$



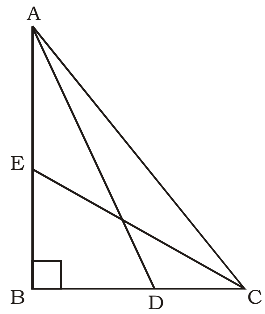
21. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

OR

21. Prove that one and only one out of n, $n + 2$ or $n + 4$ is divisible by 3, where n is any positive integer.

22. If α and β are the zeros of the quadratic polynomial $f(x) = kx^2 + 4x + 4$ such that $\alpha^2 + \beta^2 = 24$, find the values of k.

23. ΔABC is right angled at B. AD and CE are the two medians drawn from A and C respectively. If AC = 5cm, AD = $\frac{3\sqrt{5}}{2}$ cm, find the length of CE.



24. The distribution below gives the weight of 30 students of a class. Find the median weight of students.

Weight (in Kg.)	40 - 45	45 - 50	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75
No. of Students	2	3	8	6	6	3	2

OR

24. The mean of the following frequency distribution in 25. Determine the value of P:

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	18	15	P	6

25. If $\tan\theta = \frac{12}{13}$, evaluate $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

26. x takes three hours more than y to walk 30 km. But if x doubles his speed, he is ahead of y by $1\frac{1}{2}$ hours. Find their speed of walking.

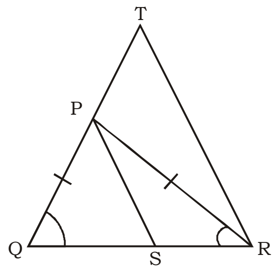
27. Without using trigonometric tables evaluate :

$$2\left[\frac{\cos 58^\circ}{\sin 32^\circ}\right] - \sqrt{3}\left[\frac{\cos 38^\circ \operatorname{cosec} 52^\circ}{\tan 15^\circ \tan 60^\circ \tan 75^\circ}\right]$$

OR

Prove that : $\cos\theta\sin\theta - \frac{\sin\theta\cos(90^\circ-\theta)\cos\theta}{\sec(90^\circ-\theta)} - \frac{\cos\theta\sin(90^\circ-\theta)\sin\theta}{\operatorname{cosec}(90^\circ-\theta)} = 0.$

28. In the figure given below, if $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle PQR = \angle PRQ$. Prove that $\Delta PQS \sim \Delta TQR$.



SECTION - D

Question numbers 29 to 34 carry 4 marks each.

29. $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

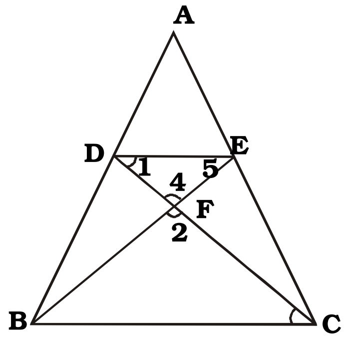
30. Obtain all other zeroes of $2x^4 - 6x^3 + 3x^2 + 3x - 2$, if two of it's zeroes are $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$.

OR

30. In trapezium ABCD, AB || DC, DC = 2AB. EF || AB where E and F lie on BC and AD respectively such that $\frac{BE}{EC} = \frac{4}{3}$. Diagonal DB intersects EF at G.Prove that $7EF = 11\,AB$.

31. In fig, DE || BC and AD : DB = 5: 4,

Find $\frac{\text{Area}(\Delta DEF)}{\text{Area}(\Delta CFB)}$.



32. 2 women and 5 men together can finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

33. Solve the equations graphically :

$$2x + y = 2$$

$$2y + x = 4$$

What is the area of triangle formed by the two lines and x - axis.

34. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

OR

Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8 for some integer m.

All the Best

CBSE X Date :	MT EDUCARE LTD. SUBJECT : **MATHEMATICS** **SUMMATIVE ASSESSMENT - 1** **MODEL ANSWER PAPER**	**Set - B** Marks : 90 Time : 3 hrs.

Any method mathematically correct should be given full credit of marks.

SECTION - A

1. (b) a rational number
2. (a) a = 2b
3. (b) 8 cm
4. (a) $x^2 - 9$
5. (c) Ogive
6. (d) 6
7. (c) 19
8. (d) cos A
9. (b) 20°, 30°
10. (d) $\frac{7}{17}$

SECTION - B

11. Given, $x = 60$, $f_1 = 11$, $f_0 = 6$, $f_2 = 8$, h = 10

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 60 + \left[\frac{11-6}{2\times11-6-8}\right] \times 10$$

$$= 60 + \frac{50}{6} = 66.25$$

12. On dividing $3x^4 + 5x^3 - 7x^2 + 2x + 2$ by $x^2 + 3x + 1$

$$
\begin{array}{l}
x^2 + 3x + 1 \overline{)\, 3x^4 + 5x^3 - 7x^2 + 2x + 2} \,(3x^2 - 4x + 2 \\
\qquad\qquad 3x^4 + 9x^3 + 3x^2 \\
\qquad\qquad - \quad - \quad - \\
\hline
\qquad\qquad\quad -4x^3 - 10x^2 + 2x + 2 \\
\qquad\qquad\quad -4x^3 - 12x^2 - 4x \\
\qquad\qquad\quad + \quad + \quad + \\
\hline
\qquad\qquad\qquad\quad 2x^2 + 6x + 2 \\
\qquad\qquad\qquad\quad 2x^2 + 6x + 2 \\
\qquad\qquad\qquad\quad - \quad - \quad - \\
\hline
\qquad\qquad\qquad\qquad 0
\end{array}
$$

Reminder is 0 hence $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

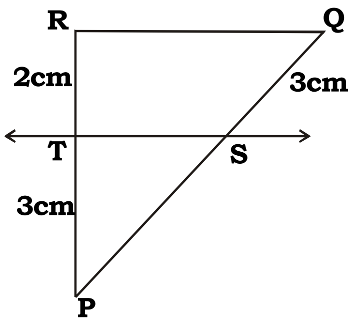
13. If 6^n ends with digit zero, then it will be divisible by 5, i.e., the prime factorisation of 6^n. must contain the prime number 5.
This is not possible because $6^n = (2 \times 3)^n = 2^n \times 3^n$
This shows that the only prime factorisation of 6^n are 2 and 3 by uniqueness
fundamental theorem of Arithmetic, there are no other primes in the factorisation of 6^n. So there is no natural number n for which 6^n ends with digit zero.

14. In ΔPRQ, we have
$ST \parallel QR$

$$\Rightarrow \frac{PS}{QS} = \frac{PT}{RT}$$

$$\Rightarrow \frac{PS}{3} = \frac{3}{2}$$

$$\Rightarrow PS = \frac{9}{2} cm = 4.5 \text{ cm}$$



15.
$$\sin (A + B) = \frac{\sqrt{3}}{2} = \sin 60°$$

$$A + B = 60° \quad ....(i)$$

and
$$\cos (A - B) = \frac{\sqrt{3}}{2} = \cos 30°$$

$$\Rightarrow \quad A - B = 30° \quad ... (ii)$$

On adding (i) (ii), We get

$$2A = 90°$$

i.e.,
$$A = 45°$$

Putting the value of A in eq. (i), we get

$$B = 15°$$

OR

15. $\because$
$$A + B + C = 180°$$
$$A + B = 180° - C$$

$$\text{L.H.S.} = \cos\left(\frac{A+B}{2}\right) = \cos\left(\frac{180° - C}{2}\right)$$

$$= \cos\left(90^\circ - \frac{C}{2}\right)$$

$$= \sin\frac{C}{2}$$

16. $15 = 3 \times 5$

$18 = 2 \times 3^2$

$45 = 5 \times 3^2$

So, H.C.F. $= 3$

L.C.M $= 3^2 \times 2 \times 5 = 90$.

17. Let us assume that $15 + 17\sqrt{3}$ is a rational number.

$$15+17\sqrt{3} = \frac{p}{q}$$

$$17\sqrt{3} = \frac{p}{q} - 15$$

$$\sqrt{3} = \frac{p-15q}{17q}$$

Since p and q are integers

$\frac{p-15q}{17q}$ is a rational number

$\therefore$ $\sqrt{3}$ is rational

But we know that $\sqrt{3}$ is irrational.

our assumption is wrong,

$\therefore$ $15 + 17\sqrt{3}$ is irrational.

18. The given system of equation is

$2x - y - 3 = 0$

$4x + y - 3 = 0$

By cross- multiplication, we get

$$\frac{x}{\begin{matrix} -1 & \searrow\!\!\!\swarrow & -3 \\ 1 & & -3 \end{matrix}} = = \frac{-y}{\begin{matrix} 2 & \searrow\!\!\!\swarrow & -3 \\ 4 & & -3 \end{matrix}} = \frac{1}{\begin{matrix} 2 & \searrow\!\!\!\swarrow & -1 \\ 4 & & 1 \end{matrix}}$$

$\Rightarrow \quad \dfrac{x}{1\times -3 - 1\times -3} = \dfrac{-y}{2\times -3 - 4\times -3} = \dfrac{1}{2\times 1 - 4\times -1}$

$\Rightarrow \quad \dfrac{x}{3 + 3} = \dfrac{-y}{-6+12} = \dfrac{1}{2 + 4}$

$\Rightarrow \quad \dfrac{x}{6} = \dfrac{-y}{-6} = \dfrac{1}{6}$

$\Rightarrow \quad x = \dfrac{6}{6} -1$ and $y = -\dfrac{6}{6} = -1$

Hence, the solution of the given system of equations is x = 1, y = –1.

SECTION - C

Question numbers 15 to 24 carry 3 marks each.

19. Suppose $\dfrac{1}{2}-\dfrac{\sqrt{5}}{3}$ is rational.

$$\frac{1}{2}-\frac{\sqrt{5}}{3}=\frac{p}{q}, \quad q\neq 0$$

$$\sqrt{5}=\frac{3q-6p}{2q}, \quad q\neq 0$$

$\sqrt{5}$ is irrational while $\dfrac{3q-6p}{2q}$ is rational abd an irrational number can never be equal to a rational number. Thus our assumption is wrong.

Hence $\dfrac{1}{2}-\dfrac{\sqrt{5}}{3}$ is irrational.

20. In ΔPOR, $\angle$ POR = 90° so by Pythagoras theorem,

$PR^2 = PO^2 + OR^2$

According to question OP = 6 sm, OR = 8 cm and QR = 26 cm

$PR^2 = 6^2 + 8^2$

$PR^2 = 10^2 \Rightarrow PR = 10$

In the right triangled QPR by Pythagoras theorem,

$QR^2 = PQ^2 + PR^2$

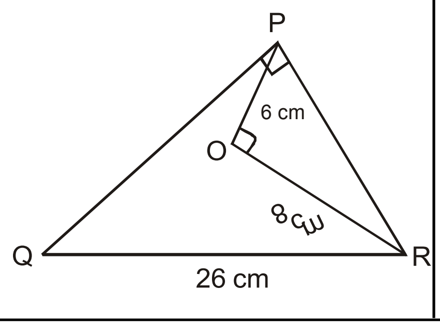
$PQ^2 = 26^2 - 10^2$

$= 676 - 100$

$= 576$

Hence PQ $= \sqrt{576}$

= 24 m



$$\text{ar}(\Delta PQR) = \frac{1}{2} \times PR \times PQ$$
$$= \frac{1}{2} \times PR \times PQ$$
$$= \frac{1}{2} \times 10 \times 24 = 120 \text{ cm}^2$$

21. $n^3 - n = n(n^2 - 1) = n(n + 1)(n - 1) = (n - 1)n(n + 1)$

= product of three consecutive positive integers.

Now, we have to show that the product of three consecutive positive integers is divisible by 6.Let a, a + 1, a + 2 by any three consecutive integers a.
Let a, a + 1, a + 2 by any three consecutive integers.

Case I. If a = 3q.

$a(a + 1)(a + 2) = 3q(3q + 1)(3q + 2)$
$= 3q$ (even number, say 2r) $= 6qr$,

($\therefore$ Product of two consecutive integers (3q + 1) and (3q + 2) is an even integer which is divisible by 6.)

Case II. If a = 3q + 1

$a(a + 1)(a + 2) = (3q + 1)(3q + 2)(3q + 3)$
= (even number, say 2r) (3) (q + 1)
= 6 (rq + r), which is divisible by 6.

Case III. If a = 3q + 2.

$a(a + 1)(a + 2) = (3q + 2)(3q + 3)(3q + 4)$
= multiple of 6 for every q
= 6r (say), which is divisible by 6.

Hence, the product of three consecutive integers is divisible by 6.

OR

21. We Know that any positive integer is of the form 3q, 3q + 1 or 3q + 2 for some integer q.

Case I : when $n = 3q$,

$n = 3q + 0 \Rightarrow n$ is divisible by 3
$n + 2 = 3q + 2 \Rightarrow n + 2$ is not divisible by 3.
and $n + 4 = 3q + 4 = 3(q + 1) + 1 \Rightarrow n + 4$ is not divisible by 3.

Case II : when $n = 3q + 1$,

$n = 3q + 1 \Rightarrow n$ is divisible by 3
$n + 2 = (3q + 1) + 2 = 3(q + 1) + 0$

Here remainder is zero, so $(n + 2)$ is divisible by 3
and $n + 4 = (3q + 1) + 4 = 3(q + 1) + 2$
$\Rightarrow (n + 4)$ is not divisible by 3.

Case III : when $n = 3q + 2$.

$n = 3q + 2 \Rightarrow$ is not divisible by 3

$n + 2 = (3q + 2) + 2 = 3(q + 1) + 1$

$\Rightarrow$ $n + 2$ is not divisible by 3

and $n + 4 = (3q + 2) + 4 = 3(q + 2) + 0$

Here remainder is zero, so $(n + 4)$ is divisible by 3.

Thus, we conclude that one and only one out of n, $n + 2$ and $n + 4$ is divisible by 3.

22. Since a and b are the zeros of the quadratic polynomial

$f(x) = kx^2 + 4x + 4$

$\therefore \quad \alpha + \beta = -\frac{4}{k}$ and $\alpha\beta = \frac{4}{k}$

Now, $\alpha^2 + \beta^2 = 24$

$\Rightarrow \quad (\alpha+\beta)^2 - 2\alpha\beta = 24$

$\Rightarrow \quad \left(-\frac{4}{k}\right)^2 - 2 \times \frac{4}{k} = 24$

$\Rightarrow \quad \frac{16}{k^2} - \frac{8}{k} = 24$

$\Rightarrow \quad 16 - 8k = 24k^2$

$\Rightarrow \quad 3k^2 + k - 2 = 0$

$\Rightarrow \quad 3k(k+1) - 2(k+1) = 0$

$\Rightarrow \quad (k+1)(3k - 2) = 0$

$\Rightarrow \quad k+1 = 0$ or, $k = \frac{2}{3}$

Hence, $k = -1$ or, $k = \frac{2}{3}$

23. By pythagoras theorem, In ΔABD,

$AB^2 + BD^2 = AD^2$

$AC^2 - BC^2 + BD^2 = AD^{23}$

$AC^2 - AD^2 = BC^2 - BD^2$

In ΔBEC, $\quad 5^2 - \left(\frac{3\sqrt{5}}{2}\right)^2 = CE^2 - BE^2 - BD^2$

$$25 - \frac{45}{4} = CE^2 - \frac{AB^2}{4} - \frac{BC^2}{4}$$

$$25 - \frac{45}{4} = CE^2 - \frac{1}{4}(AB^2 + BC^2)$$

$$= CE^2 - \frac{1}{4} \times 25$$

$$CE^2 = \frac{100 - 45 + 25}{4} = 20$$

$$CE = 2\sqrt{5} \text{ cm}$$

24.

Weight (in kg.)	**No. of students**	**c.f.**
40 – 45	2	2
45 – 50	3	5
50 – 55	8	*c.f.* = 13
l = 55 – 60	*f* = 6	19
60 – 65	6	25
65 – 70	3	28
70 – 75	2	30
	n = 30	

$$\frac{n}{2} = 15$$

$$\text{Medain} = l + \left(\frac{\frac{n}{2} - c.f.}{f}\right) \times h$$

$$= 55 + \left(\frac{15 - 13}{6}\right) \times 5$$

$$= 55 + 1.67$$

$$= 56.67$$

OR

24.

C.I.	x_i	f_i	$f_i x_i$
0 - 10	5	5	25
10 - 20	15	18	270
20 - 30	25	15	375
30 - 35	35	p	$35p$
40 - 50	45	6	270
		$\Sigma f_i = 44 + p$	$\Sigma f_i x_i = 940 + 35p$

$$\text{Mean} = \frac{\Sigma fixi}{\Sigma fi} = \frac{940 + 35p}{44 + p}$$

$$25 = \frac{940 + 35p}{44 + p}$$

$$940 + 35P = 1100 + 25p$$
$$10P = 160$$
$$P = 16$$

25. We have,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{\frac{2\sin\theta\cos\theta}{\cos^2\theta}}{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$= \frac{2 \times \frac{12}{13}}{1 - \left(\frac{12}{13}\right)^2} = \frac{\frac{24}{13}}{1 - \frac{144}{169}} = \frac{\frac{24}{13}}{\frac{25}{169}} = \frac{24}{13} \times \frac{169}{25} = \frac{312}{25}$$

26. Let speed of x be p km/h
and speed of y be q km/h

$$\text{Time taken by } x = \frac{30}{p}$$

$$\text{Time taken by y} = \frac{30}{q}$$

By question, $$\frac{30}{p} = \frac{30}{q} + 3 \qquad \text{......(i)}$$

If Speed of x is doubled it becomes $2p$, then

$$\frac{30}{2p} + \frac{3}{2} = \frac{30}{q} \qquad \text{.......(ii)}$$

Let $\frac{1}{p} = a$, $\frac{1}{q} = \text{b}$

$\therefore$ $30a = 30b + 3$

$\Rightarrow$ $10a - 10b = 1$

$\Rightarrow$ $a - b = \frac{1}{10}$ ....(iii)

and (ii) is $15a + \frac{3}{2} = 30b$

$15a - 30b = \frac{-3}{2}$

$5a - 10b = \frac{-1}{2}$

$a - 2b = \frac{-1}{10}$ ....(iv)

Now (iii) - (iv) given, $b = \frac{2}{10} = \frac{1}{5}$

$\frac{1}{q} = \frac{1}{5} \Rightarrow q = 5\text{km/hr}$

From (iii), we get $a = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$

$\Rightarrow$ $\frac{1}{p} = \frac{3}{10}$

$\Rightarrow$ $p = \frac{10}{3} = 3\frac{1}{3}$ km/h

27. $2\left[\frac{\cos 58°)}{\sin 32°}\right] - \sqrt{3}\left[\frac{\cos 38° \operatorname{cosec} 52°}{\tan 15° \tan 60° \tan 75°}\right] = 2\left[\frac{\sin(90° - 58°)}{\sin 32°}\right] - \sqrt{3}\left[\frac{\sin(90° - 38°) \operatorname{cosec} 52°}{\tan 15° \times \sqrt{3} \times \cot(90° - 75°)}\right]$

$= 2\left[\frac{\sin 32°}{\sin 32°}\right] - \sqrt{3}\left[\frac{\sin 52° \times \operatorname{cosec} 52°}{\tan 15° \times \sqrt{3} \times \cot 15°}\right]$

$= 2 - \sqrt{3}\left[\frac{\sin 52° \times \frac{1}{\sin 52°}}{\tan 15° \times \sqrt{3} \times \frac{1}{\tan 15°}}\right]$

$= 2 - \frac{\sqrt{3}}{\sqrt{3}}$

$= 2 - 1 = 1.$

OR

$$\textbf{L.H.S} = \cos\theta\sin\theta - \frac{\sin\theta\cos(90^\circ-\theta)\cos\theta}{\sec(90^\circ-\theta)} - \frac{\cos\theta\sin(90^\circ-\theta)\sin\theta}{\operatorname{cosec}(90^\circ-\theta)}$$

$$= \cos\theta\sin\theta - \frac{\sin\theta\sin\theta\cos\theta}{\operatorname{cosec}\theta} - \frac{\cos\theta\sin(90^\circ-\theta)\sin\theta}{\operatorname{cosec}(90^\circ-\theta)}$$

$$= \cos\theta\sin\theta - \frac{\sin^2\theta\cos\theta}{1/\sin\theta} - \frac{\cos^2\theta\sin\theta}{1/\cos\theta}$$

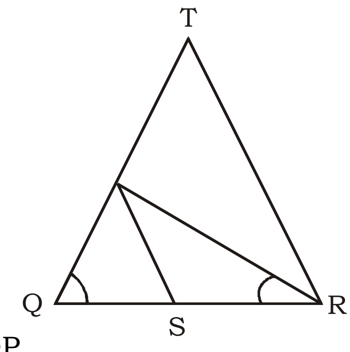
$$= \cos\theta\sin\theta - \sin^3\theta\cos\theta - \cos^3\theta\sin\theta$$

$$= \cos\theta\sin\theta - (\sin\theta\cos\theta)(\sin^2\theta + \cos^2\theta)$$

$$= \sin\theta\cos\theta - \sin\theta\cos\theta = 0$$

= **R.H.S.**

28. Given, $\frac{QR}{QS} = \frac{QT}{PR}$



$\because$ $\angle PQR = \angle PRQ$

$\therefore$ $PQ = PR$

From (1), $\frac{QR}{QS} = \frac{QT}{PQ} \Rightarrow \frac{QS}{QR} = \frac{PQ}{QT}$

In ΔPQS and ΔTQR, $\angle Q$ is common and $\frac{QS}{QR} = \frac{QP}{QT}$

$\therefore$ By SAS, $\Delta PQS \sim \Delta TQR$.

SECTION D

29. L.H.S $= (\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$$

$$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$$

$$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$$

$$= \mathbf{\cos A \cdot \sin A} \qquad \textbf{.......... (1)}$$

R.H.S. $= \frac{1}{\tan A + \cot A}$

$$= \frac{1}{\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)}$$

$$= \frac{1}{\left(\frac{\sin^2 A + \cos^2 A}{\cos A - \sin A}\right)}$$

$$= \frac{\cos A . \sin A}{\sin^2 A + \cos^2 A}$$

$= \mathbf{\cos A . \sin A}$ .... **(2)** .... $(\because \sin^2 A + \cos^2 A = 1)$

From (1) and (2),

L.H.S. = R.H.S.

30. Two zeroes are $\frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}$

One factor is $\left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right)$

i.e.,

$2x^2 - 1) \overline{2x^4 - 6x^3 + 3x^2 + 3x - 2} (x^2 - 3x + 2$

$2x^4 \quad - x^2$

$- \quad +$

$-6x^3 + 4x^2 + 3x$

$-6x^3 \quad + 3x$

$+ \quad -$

$4x^2 - 2$

$4x^2 - 2$

$- \quad +$

0

Another factor is $x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = +1$ and $+2$

Hence, other zeroes are 1 and 2.

OR

30. In trapezium ABCD, $AB \| DC$ and DC = 2AB.

Also $\frac{BE}{EC} = \frac{4}{3}$

In trapezium ABCD, $EF \| AB \| CD$

$$\frac{AF}{FD} = \frac{BE}{BC} = \frac{4}{3}$$

In ΔBGE and ΔBDC,

$$\angle B = \angle B$$
$$\angle BEG = \angle BCD$$
$$\Delta BGE \sim \Delta BDC$$

$\therefore$ $$\frac{EG}{CD} = \frac{BE}{BC}$$

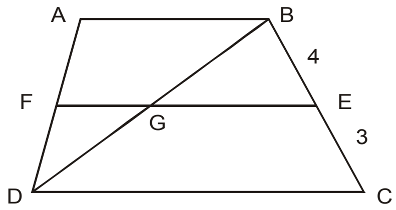
As $\frac{BE}{EC} = \frac{4}{3}$ $\Rightarrow$ $\frac{EC}{BE} = \frac{3}{4}$

$\Rightarrow$ $$\frac{EC}{BE} + 1 = \frac{3}{4} + 1$$

$\Rightarrow$ $$\frac{EC + BE}{BE} = \frac{7}{4}$$

$\Rightarrow$ $$\frac{BC}{BE} = \frac{7}{4}$$

$\Rightarrow$ $$\frac{BE}{BC} = \frac{4}{7}$$



$\Rightarrow$ $$\frac{EG}{CD} = \frac{4}{7}$$

$\Rightarrow$ $$EG = \frac{4}{7} CD$$

Similarly $\Delta DGF \sim \Delta DBA$ $\Rightarrow$ $\frac{DF}{DA} = \frac{FG}{AG}$

$\Rightarrow$ $$\frac{FG}{AB} = \frac{3}{7}$$

$\Rightarrow$ $$FG = \frac{3}{7} AB$$

$$\left[\begin{array}{l} \because \frac{AF}{FD} = \frac{4}{7} = \frac{BE}{BD} \\ \Rightarrow \frac{EC}{BC} = \frac{3}{7} \end{array}\right]$$

Adding (i) and (ii), we get $EG + FG = \frac{4}{7} CD + \frac{3}{7} AB$

$$EF = \frac{4}{7} \times (2AB) + \frac{3}{7} AB = \frac{8}{7} AB + \frac{3}{7} AB = \frac{11}{7} AB$$

$\Rightarrow$ $7EF = 11 AB.$

31. In DABC, we have

DE || BC

$\Rightarrow \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$ [Corresponding angles]

Thus, in triangles ADE and ABC, we have

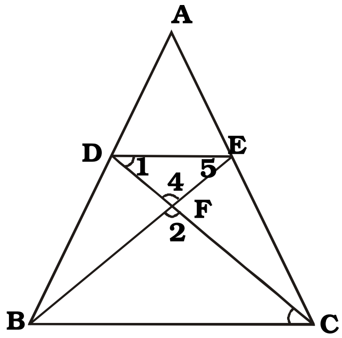
$\angle A = \angle A$ [Common]

$\angle ADE = \angle ABC$

$\Rightarrow \angle AEE = \angle ACB$

$\therefore \Delta AED \sim \Delta ABC$ [By AAA similarity]

$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC}$$



We have,

$$\frac{AD}{BD} = \frac{5}{4}$$

$$\Rightarrow \frac{DB}{AD} = \frac{4}{5}$$

$$\Rightarrow \frac{DB}{AD} + 1 = \frac{4}{5} + 1$$

$$\Rightarrow \frac{DB + AD}{AD} = \frac{9}{5}$$

$$\Rightarrow \frac{AB}{AD} = \frac{9}{5} \Rightarrow \frac{AD}{AB} = \frac{5}{9}$$

$$\therefore \frac{DE}{BC} = \frac{5}{9}$$

In ΔDFE and ΔCFB, we have

$\angle 1 = \angle 3$ [Alternate interior angles]

$\angle 2 = \angle 4$ [Vertically opposite angles]

Therefore, by AA-similarity criterion,we have

$\Delta DFE \sim \Delta CFB$

$$\Rightarrow \frac{\text{Area}(\Delta DEF)}{\text{Area}(\Delta CFB)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \frac{\text{Area}(\Delta DEF)}{\text{Area}(\Delta CFB)} = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$$

32. Let the no. of days a woman takes to complete the work alone be x & that a man would take be y.

$\therefore$ Work done by 1 woman in one day $= \frac{1}{x}$

$\therefore$ Work done by 1 man in one day = $\frac{1}{y}$

According to the first condition,

$$\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$$

According to the second condition,

$$\frac{3}{x} + \frac{6}{y} = \frac{1}{3}$$

Substitute $\frac{1}{x} = p$ & $\frac{1}{y} = q$

$\therefore$ eq^n (i) and eq^n (ii) reduce to,

$\therefore \quad 2p + 5q = \frac{1}{4}$

$\therefore \quad 8p + 20q = 1$

$\therefore \quad 8p = 1 - 20q$

$\therefore \quad p = \frac{1 - 20q}{8}$ ...... (i)

$3p + 6q = \frac{1}{3}$

$9p + 18q = 1$ ...... (ii)

Substituting eq^n (i) in eq^n (ii)

$\therefore 9\left(\frac{1-20q}{8}\right) + 18q = 1$

$\therefore \frac{9-180q}{8} + 18q = 1$

$\therefore 9 - 180q + 144q = 8$

$\therefore \quad -36q = -1$

$\therefore \quad \mathbf{q = \frac{1}{36}}$

Substituting $q = \frac{1}{36}$ in eq^n (i) ,

$p = \frac{1-20\left(\frac{1}{36}\right)}{8}$

$p = \frac{\frac{1}{1} - \frac{20}{36}}{8}$

$= \frac{36 - 20}{36 \times 8}$

$= \frac{16}{36 \times 8}$

$\therefore \quad \boldsymbol{p = \frac{1}{18}}$

Resubstituting for p and q.

$p = \frac{1}{x}$

$\therefore \quad \frac{1}{18} = \frac{1}{x}$

$\therefore \quad \boldsymbol{x = 18}$

$q = \frac{1}{y}$

$\therefore \quad \frac{1}{36} = \frac{1}{y}$

$\therefore \quad \boldsymbol{y = 36}$

$\therefore$ **Woman would take 18 days to complete the work alone and a Man will take 36 days to complete the work alone.**

33. 27. $2x + y = 2 \Rightarrow x = \frac{2-y}{2}$

Table of this equation (i) is .....(i)

x	1	0	2
y	0	2	-2

$$2y - x = 4$$

$$\Rightarrow \quad y = \frac{x+4}{2}$$

Table of this equation (ii) is .....(ii)

x	0	2	4
y	2	3	4

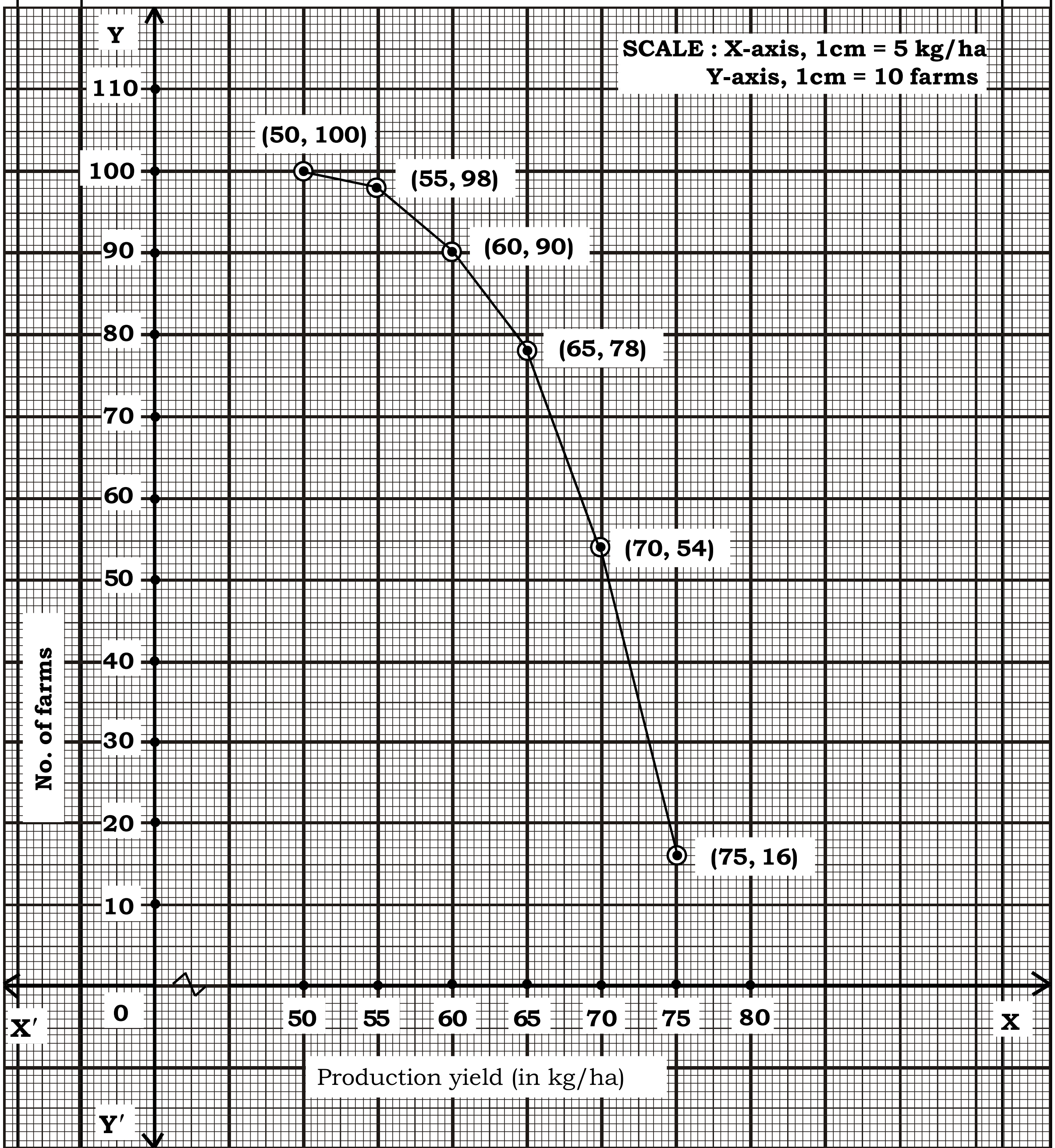
$$Ar(\Delta ABC) = \frac{1}{2} \times BC \times AO$$

$$= \frac{1}{2} \times 5.5 \times 2$$

$$= 5.5 \text{ cm}^2$$

34.

Production yield (in kg/ha.)	Number of farms	Production yield	Cumulative Frequency	Points to be plotted
50 - 55	2	50 or more than 50	100	(50, 100)
55 - 60	8	55 or more than 55	98	(55, 98)
60 - 65	12	60 or more than 60	90	(60, 90)
65 - 70	24	65 or more than 65	78	(65, 78)
70 - 75	38	70 or more than 70	54	(70, 54)
75 - 80	16	75 or more than 75	16	(75, 16)



Points are (120, 12), (140, 26), (160, 34), (180, 40); (200, 50)

OR

34. Let x be any positive integer and b = 3

Applying Euclid's Division Algorithm

$\therefore\ x = 3q + r$ where $0 \leq r < 3$

The possible remainders are 0, 1, 2

$\therefore\ x = 3q$ or $3q + 1$ or $3q + 2$

i) If $x = 3q \Rightarrow x^3 = (3q)^3 = 27q^3 = 9(3q^3) = \mathbf{9m}$ for some integer m, where $m = 3q^3$

ii) If $x = 3q + 1 \Rightarrow x^3 = (3q+1)^3 = (3q)^3 + 3(3q)^2(1) + 3(3q)(1)^2 + (1)^3$

[$\because$ since $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= 27q^3 + 27q^2 + 9q + 1$

$= 9q(3q^2 + 3q + 1) + 1$

$= \mathbf{9m + 1}$ for some integer m, where $m = q(3q^2 + 3q + 1)$

iii) If $x = 3q + 2 \Rightarrow x^3 = (3q + 2)^3$

$= (3q)^3 + 3(3q)^2(2) + 3(3q)(2)^2 + (2)^3$

[$\because$ since $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$]

$= 27q^3 + 54q^2 + 36q + 8$

$= 9q(3q^2 + 6q + 4) + 8$

$= \mathbf{9m + 8}$ for some integer m, where $m = q(3q^2 + 6q + 4)$

cube of any positive integer is either of the form 9*m*, 9*m* + 1 or 9*m* + 8

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