MT EDUCARE LTD.

SUMMATIVE ASSESSMENT - 1

CBSE - X **2013-14** **Set - A**

Roll No. | | | | | | | |

Code No. 31/1

Series RLH

- Please check that this question paper contains 6 printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains 34 questions.
- Please write down the serial number of the question before attempting it.

MATHEMATICS

Time allowed **:** 3 hours ***Maximum Marks*** **:** 80

General Instructions:

i) All questions are compulsory.

ii) The question paper consists of 34 questions divided in four sections: A,B,C and D.
Section **A** comprise 10 questions of 1 mark each,
Section **B** comprise 8 questions of 2 marks each,
Section **C** comprise 10 questions of 3 marks each, and
Section **D** comprise 6 questions of 4 marks each.

iii) Question numbers 1 to 10 in Section A are multiple choice questions where you have to select one correct option out of the given four.

iv) There is no overall choice. However, internal choice has been provided in 1 question of two marks, 3 questions of three marks each and 2 questions of four marks each. You have to attempt only of the alternative in all such questions.

v) Use of calculator is not permitted.

SECTION - A

Question number 1 to 10 carry 1 marks each.

1. If a pair of linear equations in two variables is consistent, then the lines respresented by two equations are

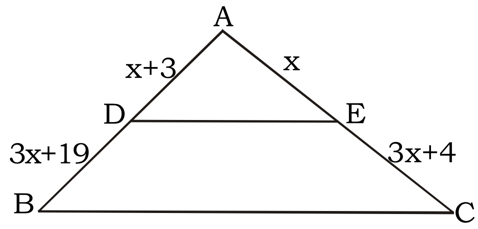
   (a) intersecting (b) parallel

   (c) always coincident (d) intersecting or coincident

2. In fig. the value of x for which DE || AB is

   (a) 4 (b) 1

   (c) 3 (d) 2



3. If $\tan^2 45° - \cos^2 30° = x \sin 45° \cos 45°$, then x =

   (a) 2 (b) -2 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

4. If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3q$; p, q being prime numbers, then LCM (a,b) is

   (a) pq (b) p^3q^3 (c) p^3q^2 (d) p^2q^2

5. If the mean of a frequency distributio is 8.1 and $\sum fixi = 132 + 5k$, $\sum fi = 20$, then k =

   (a) 3 (b) 4 (c) 5 (d) 6

6. If α, β are the zeros of the polynomial $p(x) = 4x^2 + 3x + 7$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is equal to

   (a) $\frac{7}{3}$ (b) $-\frac{7}{3}$ (c) $\frac{3}{7}$ (d) $-\frac{3}{7}$

7. $9 \sec^2 A - 9 \tan^2 A$ is equal to

   (a) 1 (b) 9 (c) 8 (d) 0

8. Which of the following rational numbers have terminating decimal ?

   (a) $\frac{16}{225}$ (b) $\frac{5}{18}$ (c) $\frac{2}{21}$ (d) $\frac{7}{250}$

9. If x + 2 is a factor of $x^2 + ax + 2b$ and a + b = 4, then
(a) a = 1, b = 3 (b) a = 3, b = 1 (c) a = -1, b = 5 (d) a = 5, b = -1

10. If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by
(a) 5 (b) 13
(c) both 5 and 13 (d) None of these

SECTION - B

Question number 11 to 18 carry 2 marks each.

11. For the following grouped frequency distribution find the mode :

Class	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	5	10	23	21	12	3

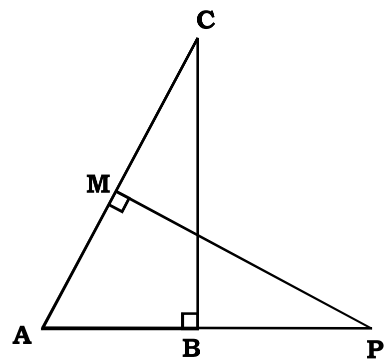
12. Check whether $x^3 - 3x + 1$ is a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

13. Solve the each of the following system of equations by using the method of cross-multiplication: x +y = 7 , 5x + 12y – 7 = 0

14. In a right triangle ABC, right angled at C, if tan A = 1, then verify that 2sinA cos A = 1.

Or

14. Given :
ΔABC & ΔAMP are two right triangles, right angled at B & M respectively.
Prove that :
i) $\Delta ABC \sim \Delta AMP$
ii) $\frac{CA}{PA} = \frac{BC}{MP}$



15. Given that HCF (306, 657) = 9, find LCM (306, 657)

16. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

17. Given 15 cot A = 8, find sin A and sec A

18. Solve the following pair of linear equations by the substitution method.
$0.2x + 0.3y = 1.3$; $0.4x + 0.5y = 2.3$

SECTION - C

Question numbers 19 to 28 carry 3 marks each.

19. The sum of the digits of a two digit number is 8 and the difference between the number and that formed by reversing the digits is 18. Find the number.

20. If α,β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation
$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible.

21. Find the values of α and β for which the following system of linear equations has infinite number of solutions:
$2x + 3y = 7$
$2\alpha x + (\alpha+\beta) y = 28$

OR

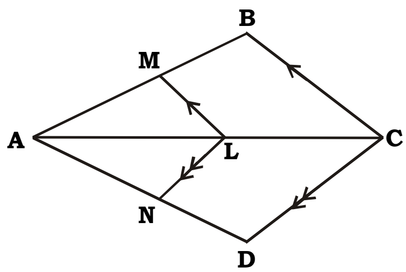
21. Find the zeroes of the following quadratic polynomials and verify $t^2 - 15$.

22. Evaluate the following :

$$\frac{5\cos^2 60^0 + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^0 + \cos^2 30^0}$$

23. Given : LM || CB
LN || CD

Prove that : $\frac{AM}{AB} = \frac{AN}{AD}$

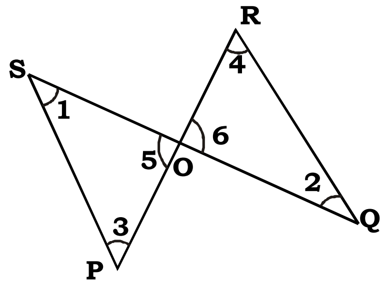


24. Prove the following identities where the angles involved are acute angles for which the expressions are defined.
$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

OR

24. In ΔOPQ right angled at P, OP = 7cm, OQ – PQ = 1cm. Determine the values of sin Q and cosQ.

25. In fig. if Δ POS ~ ΔROQ, prove that PS || QR.



26. Prove that $\sqrt{3}$ is an irrational number.

27. Find the LCM and HCF of the following pairs 336 and 54.

28. Solve the following system of linear equations graphically:

$x - y = 1$
$2x + y = 8$
Shade the area bounded by these two lines and y-axis.

OR

Prove the following :

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \cos A + \sin A$$

SECTION - D

Question numbers 29 to 34 carry 3 marks each.

29. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$ for some integer m.

OR

Use Euclid's division lemma to show that the square of any positive integer either of the form 3m or $3m + 1$ for some integer m.

30. In fig., DE || BC and AD : DB = 5 : 4 Find $\frac{\text{Area}(\Delta DEF)}{\text{Area}(\Delta CFB)}$

31. During the medical check-up of 35 students of a class, their weights were recorded as, follows :

Weight (in kg)	Number of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

Draw a less than type ogive for the given data. Hence obtain the median weight from the graph and verify the result by using the formula.

32. Points A and B are 90 km apart from each other on a highway. A car starts from A and another from B at the same time. If they go in the same direction they meet in 9hours and if they go in opposite directions they meet in 9/7hours. Find their speeds.

33. If a line is drawn parallel to one side of a triangle, to intersect the other two sides in distinct points, then other two sides are divided in the same ratio.

OR

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

34. Prove the following $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$ = cosec A + cot A using the identity $\text{cosec}^2 A = 1 + \cot^2 A$.

All the Best 👍

CBSE X Date :	MT EDUCARE LTD. SUBJECT : **MATHEMATICS** **SUMMATIVE ASSESSMENT - 1** **MODEL ANSWER PAPER**	**Set - A** Marks : 80 Time : 3 hrs.

Any method mathematically correct should be given full credit of marks.

SECTION - A

1. (d) intersecting or coincident
2. (a) 4
3. (d) $\frac{1}{2}$
4. (c) p^3q^2
5. (d) 6
6. (d) $-\frac{3}{7}$
7. (b) 9
8. (d) $\frac{7}{250}$
9. b) a = 3, b = 1
10. c) both 5 and 13

SECTION - B

11. We observe that the class 12 - 15 has maximum frequency. Therefore, this the modal class

We have,

l = 12 ,h = 3 , f = 23, f_1 = 10 and f_2 = 21

$\therefore$ Mode $= l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$\Rightarrow$ Mode $= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$

$\Rightarrow$ Mode $= 12 + \frac{13}{15} \times 3 = 12 + \frac{13}{5} = 14.6$

12.

$$
\begin{array}{r|l}
 & \quad x^2 - 1 \\
\hline
x^3 - 3x + 1 & x^5 - 0x^4 - 4x^3 + x^2 + 3x + 1 \\
 & x^5 \qquad\quad - 3x^3 + x^2 \\
 & - \qquad\qquad + \qquad - \\
\hline
 & \qquad\qquad - x^3 \qquad + 3x + 1 \\
 & \qquad\qquad - x^3 \qquad + 3x - 1 \\
 & \qquad\qquad + \qquad\quad - \quad + \\
\hline
 & \qquad\qquad\qquad\qquad\qquad 2
\end{array}
$$

13. The given system of equation is

$x + y = 7$

$5x + 12y - 7 = 0$

By cross- multiplication, we get

$$\frac{x}{\begin{array}{cc} 1 & 7 \\ 12 & -7 \end{array}} = = \frac{-y}{\begin{array}{cc} 1 & -7 \\ 12 & -7 \end{array}} = \frac{1}{\begin{array}{cc} 1 & 1 \\ 5 & 12 \end{array}}$$

$$\Rightarrow \quad \frac{x}{1\times -7 - 12\times -7} = \frac{-y}{1\times -7 - 5\times -7} = \frac{1}{1\times 12 - 5\times 1}$$

$$\Rightarrow \quad \frac{x}{-7+84} = \frac{-y}{-7+35} = \frac{1}{12-5}$$

$$\Rightarrow \quad \frac{x}{77} = \frac{-y}{28} = \frac{1}{7}$$

$$\Rightarrow \quad x = \frac{77}{7} \text{ and } y = -\frac{28}{7} \quad \Rightarrow \quad x = 11 \text{ and } y = -4$$

Hence, the solution of the given system of equations is $x = 11$, $y = -4$.

14. In ΔABC, we have

$\tan A = 1$

$$\Rightarrow \quad \frac{BC}{AC} = 1$$

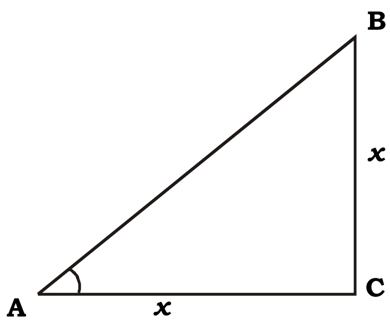
$\Rightarrow$ $BC = x$ and $AC = x$

By Pythagoras therorem, we have

$AB^2 = AC^2 + BC^2$

$\Rightarrow$ $AB^2 = x^2 + x^2$

$\Rightarrow$ $AB = \sqrt{2}x$

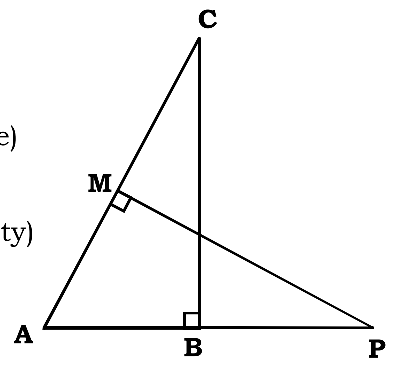


$\therefore$ $\sin A = \frac{BC}{AC} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$\Rightarrow$ $2 \sin A \cos A = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

OR

14. **Proof :**



In ΔABC & ΔAMP

$\angle BAC = \angle MAP$ ..... (common angle)

$\angle ABC = \angle AMP$ ..... (each is 90^0)

$\therefore$ $\Delta ABC \sim \Delta AMP$ ..... (by AA similarity)

$\therefore$ $\boxed{\frac{\mathbf{CA}}{\mathbf{PA}} = \frac{\mathbf{BC}}{\mathbf{MP}}}$ ..... (corresponding sides of similar triangles)

15. HCF (306, 657) = 9

Now HCF (306, 657) × LCM (306, 657) = 306 × 657

$\therefore$ LCM (306, 657) $= \frac{306 \times 657}{9}$

$= \frac{201042}{9}$

= **22338**

16. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

We have,

LHS $= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$

$\Rightarrow$ LHS $= \frac{1-\sin\theta+1-\sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$\Rightarrow$ LHS $= \frac{2}{1-\sin^2\theta}$

$\Rightarrow$ LHS $= \frac{2}{\cos^2\theta}$ $[\because 1 - \sin^2\theta = \cos^2\theta]$

$\Rightarrow$ LHS $= 2\sec^2\theta$ = RHS $\left[\because \sec\theta = \frac{1}{\cos\theta}\right]$

17. Given : 15 cot A = 8

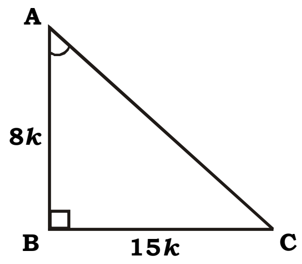
$\therefore \quad \cot A = \frac{8}{15}$ .....(i)

$\cot A = \frac{AB}{BC}$ .....(ii)

$\therefore \quad \frac{8}{15} = \frac{AB}{BC}$ .....(iii) ... [From (i) and (ii)]

$\therefore$ Let AB = $8k$, BC = $15k$ ... [where k is positive number]

In right ΔABC, using Pythagoras theorem,



$(AC)^2 = (AB)^2 + (BC)^2$

$\therefore \quad (AC)^2 = (8k)^2 + (15k)^2$

$\therefore \quad (AC)^2 = 64k^2 + 225k^2$

$\therefore \quad AC^2 = 289k^2$

$\therefore \quad AC = 17k$

$\sin A = \frac{BC}{AC} = \frac{15k}{17\,k}$

$\therefore \quad \boxed{\sin A = \frac{15}{17}}$

$\sec A = \frac{AC}{AB} = \frac{17k}{8k}$

$\therefore \quad \boxed{\sec A = \frac{17}{8}}$

18. $\therefore \quad x = \frac{1.3 - 0.3y}{0.2}$ ... (i)

$0.4\,x + 0.5\,y = 2.3$ ... (ii)

Substituting eq^n (i) in eq^n (ii)

$\therefore \quad 0.4\left(\frac{1.3 - 0.3y}{0.2}\right) + 0.5y = 2.3$

$\therefore \quad 2.6 - 0.6y + 0.5y = 2.3$

$\therefore \quad 0.3 = 0.1y$

$\therefore \quad \boldsymbol{y = 3}$

Substituting $y = 3$ in eq^n (i)

$\therefore \quad x = \frac{1.3 - 0.3(3)}{0.2}$

$= \frac{0.4}{0.2}$

$\therefore \quad \boldsymbol{x = 2}$

Solution is $x = 2$, $y = 3$

SECTION - C

19. Let the digit at unit's place be x and the digit at ten's place be y. Then,

Number = 10y + x

Number formed by reversing the digits = 10x + y

According to the given conditions, we have

$x + y = 8$

and, $(10y + x) - (10x + y) = 18$

$\Rightarrow$ $9(y - x) = 18$

$\Rightarrow$ $y - x = 2$

On solving equations (i) and (ii), we get x = 3, y = 5

Hence, number = 10y + x = 10 × 5 + 3 = 53

20. Since α and β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$.

$\therefore \quad \alpha + \beta = -\frac{5}{2}$ and $\alpha\beta = \frac{k}{2}$

Now,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$$

$$\Rightarrow (\alpha^2 + \beta^2 + 2\alpha\beta) - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow (\alpha + \beta)^2 - \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \frac{25}{4} - \frac{k}{2} = \frac{21}{4} \qquad \left[\because \alpha + \beta = -\frac{5}{2} \text{ and } \alpha\beta = \frac{k}{2}\right]$$

$$\Rightarrow -\frac{k}{2} = -1$$

$$\Rightarrow k = 2$$

21. The given system of equations will have infinite number of solution, if

$$\frac{2}{2\alpha} = \frac{3}{\alpha + \beta} = \frac{7}{28}$$

$$\Rightarrow \frac{1}{\alpha} = \frac{3}{\alpha + \beta} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{\alpha} = \frac{1}{4} \text{ and } \frac{3}{\alpha + \beta} = \frac{1}{4}$$

$$\Rightarrow \alpha = 4 \text{ and } \alpha + \beta = 12$$

$$\Rightarrow \alpha = 4 \text{ and } \beta = 8$$

Hence, the given system of equation will have infinitely many solutions, if $\alpha = 4$ and $\beta = 8$.

OR

21. $t^2 + 0t - 15$

$(t)^2 - \left(\sqrt{15}\right)^2$

$(t + \sqrt{15})\ (t - \sqrt{15})$ (using $a^2 - b^2 = (a + b)(a - b)$)

So, the value of $t^2 - 15$ is zero, When $(t + \sqrt{15}) = 0$ or $(t - \sqrt{15}) = 0$,

i.e. when $t = -\sqrt{15}$ or $t = \sqrt{15}$.

Therefore, the zeroes of $t^2 - 15$ are $-\sqrt{15}$ and $\sqrt{15}$.

Now, Sum of zeroes $= -\sqrt{15} + \sqrt{15} = 0 = \frac{-(0)}{1} = \frac{-\text{(coefficient of } t)}{\text{coefficient of } t^2}$

Product of zeroes $= -\sqrt{15} \times \sqrt{15} = \frac{-15}{1} = \frac{\text{constant term}}{\text{coefficient of } t^2}$

22. $$\frac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0} = \left[5 \times \left(\frac{1}{2}\right)^2 + 4 \times \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2\right] \div \left[\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right]$$

$$= \left(5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1\right) \div \left(\frac{1}{4} + \frac{3}{4}\right)$$

$$= \left(\frac{5}{4} + \frac{16}{3} - \frac{1}{1}\right) \div \left(\frac{1 + 3}{4}\right)$$

$$= \frac{15 + 64 - 12}{12} \div \left(\frac{4}{4}\right)$$

$$= \frac{79 - 12}{12} \times 1$$

$$= \mathbf{\frac{67}{12}}$$

23. Proof : In ΔABC,

LM $\parallel$ CB .... (given)

$\therefore \quad \frac{AM}{BM} = \frac{AL}{CL}$ ....(I) .... (by basic proportionality theorem.)

In ΔADC,

LN || CD .... (given)

$$\frac{AN}{DN} = \frac{AL}{CL} \quad ......(II) \quad .... \text{(by basic proportionality theorem)}$$

From (I) and (II)

$$\frac{AM}{BM} = \frac{AN}{DN}$$

By Invertendo (taking reciprocal)

$$\frac{BM}{AM} = \frac{DN}{AN}$$

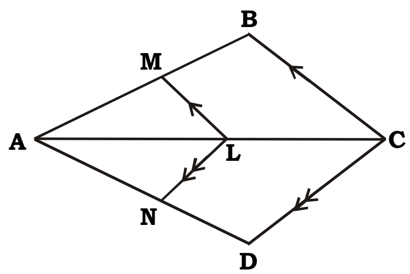
$$\frac{BM}{AM}+1 = \frac{DN}{AN}+1 \quad .... \text{(Adding 1 on both sides)}$$

$$\frac{BM+AM}{AM} = \frac{DN+AN}{AN}$$

$$\therefore \quad \frac{AB}{AM} = \frac{AD}{AN}$$

By Invertendo (taking reciprocal)

$$\therefore \quad \mathbf{\frac{AM}{AB} = \frac{AN}{AD}}$$



24. **Proof**

L.H.S = $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

= $\sin^2 A + 2\sin A \operatorname{cosec} A + \operatorname{cosec}^2 A + \cos^2 A + 2\cos A \sec A + \sec^2 A$

= $(\sin^2 A + \cos^2 A) + (\operatorname{cosec}^2 A) + (\sec^2 A) + 2\sin A \operatorname{cosec} A + 2\cos A \sec A$

= $1 + (1 + \cot^2 A) + (1 + \tan^2 A) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$

.... [$\because \sin^2 A + \cos^2 A = 1, \operatorname{cosec}^2 A = 1 + \cot^2 A, \sec^2 A = 1 + \tan^2 A$]

= $1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

= $7 + \tan^2 A + \cot^2 A$

= R.H.S.

$\therefore$ **L.H.S. = R.H.S. .... Hence proved.**

OR

24. In ΔOPQ, we have

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow$ $(PQ + 1)^2 = OP^2 + PQ^2$ [$\because OQ - PQ = 1 \Rightarrow OQ = 1 + PQ$]

$PQ^2 + 2PQ + 1 = OP^2 + PQ^2$

$\Rightarrow$ $2PQ + 1 = 49$

$\Rightarrow$ $2PQ = 48$

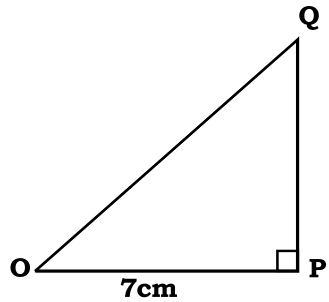
$\Rightarrow$ PQ - 24cm

$\therefore$ OQ – PQ = 1cm

$\Rightarrow$ OQ = (PQ + 1) cm = 25cm

Now $\sin Q = \frac{7}{25}$

and, $\cos Q = \frac{24}{25}$



25. We have,

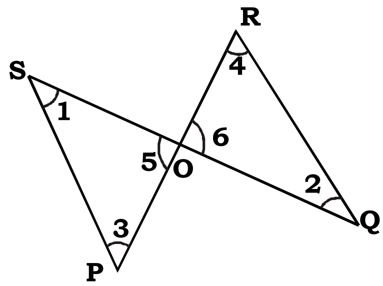
$\Delta POS \sim \Delta ROQ$

$\angle 3 = \angle 4$ and $\angle 1 = \angle 2$

Thus, PS and QR are two lines and the transversal PR cuts them in such a way that $\angle 3 = \angle 4$

i.e., alternate angles are equal.

Hence, PS || QR.



26. Let us assume that $\sqrt{3}$ is a rational number.

$\therefore$ There exist co-prime integers a and b ($\neq 0$) such that,

$\sqrt{3} = \frac{a}{b}$

$\sqrt{3}b = a$

squaring both sides,

$3b^2 = a^2$ ... (1)

$\therefore$ 3 divides a^2 $\Rightarrow$ 3 divides a ... (2)

Let $a = 3c$ where c is some integer

substituting this value of a in (1)

$3b^2 = (3c)^2$

$3b^2 = 9c^2$

$\therefore$ $b^2 = 3c^2$

$\therefore$ 3 divides b^2 $\Rightarrow$ 3 divides b ... (3)

From (2) and (3), we get,

a and b both have common factor 3. This contradicts the fact that a and b are co-prime.

$\therefore$ Our assumption that $\sqrt{3}$ is a rational number is wrong.

$\therefore$ **$\sqrt{3}$ is an irrational number.**

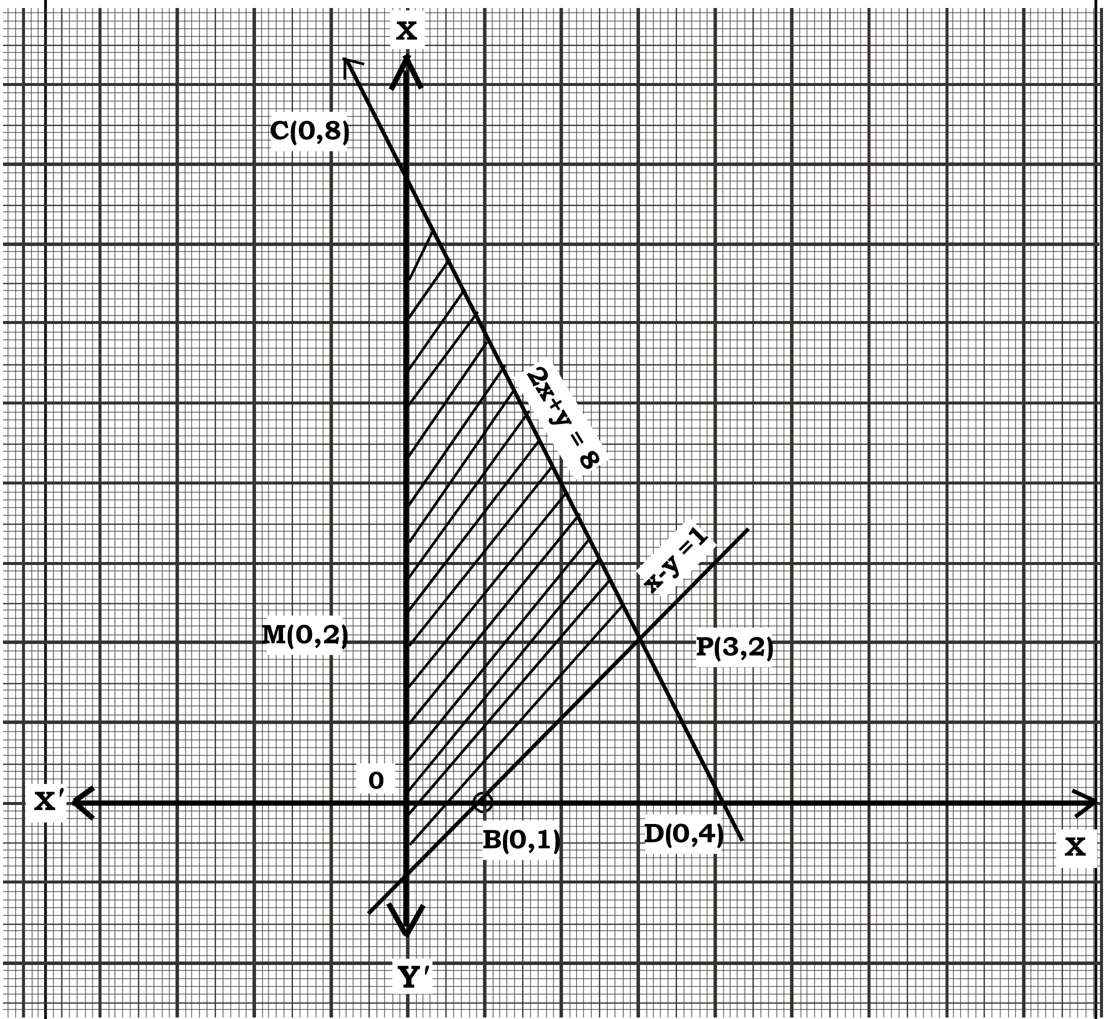
27. $336 = 2^4 \times 3 \times 7$
$54 = 2 \times 3^3$
HCF (336 and 54) = $2 \times 3 =$ **6**
(Product of common factors raised to least powers)
LCM (336 and 54) = $2^4 \times 3^3 \times 7 =$ **3024**
(Product of all the prime factors raised to highest powers)

Verification :
LCM × HCF = 6 × 3024 = **18144**
Product of the two numbers = 336 × 54 = **18144**
∴ LCM × HCF = Product of the two numbers.

28. We have,
$x - y = 1$
$2x + y = 8$
Graph of the equation $x - y = 1$:
We have,
$x - y = 1 \Rightarrow y = x - 1$ and $x = y + 1$
Putting x = 0, we get y = –1
Putting y = 0, we get x = 1
Thus, we have the following table for the points on the line $x - y = 1$:

x	0	1
y	-1	0

Plotting points A(O,–1), B(1,0) on the graph paper and drawing a line passing through them, we obtain the graph of the line reresented by the equation x –y = 1 as shown in



Graph of the equation 2x + y = 8;

We have,

$2x + y = 8 \quad \Rightarrow \quad y = 8 - 2x$ and $x = \dfrac{8-y}{2}$

Putting x = 0, we get y = 8

Putting y = 0, we get x = 4

Thus , we have the following table giving two points on the line represented by the equation 2x + y = 8.

x	0	4
y	8	0

Plotting points C (0,8) and D (4,0) on the same graph paper and drawing a line passing though them, we obtain the graph of the line represented by the equation 2x + y =8 as shown.

OR

28. we have,

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$\Rightarrow \text{LHS} = \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}}$$

$$\Rightarrow \text{LHS} = \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

$$\Rightarrow \text{LHS} = \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$\Rightarrow \text{LHS} = \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$\Rightarrow \text{LHS} = \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$\Rightarrow \text{LHS} = \frac{(\cos A - \sin A)(\cos A + \sin A)}{\cos A - \sin A}$$

$$\Rightarrow \text{LHS} = \cos A + \sin A = \text{RHS}$$

SECTION - D

29. Let x be any positive integer and b = 3

Applying Euclid's Division Algorithm

$\therefore$ $x = 3q + r$ where $0 \leq r < 3$

The possible remainders are 0, 1, 2

$\therefore$ $x = 3q$ or $3q + 1$ or $3q + 2$

i) If $x = 3q \Rightarrow x^3 = (3q)^3 = 27q^3 = 9(3q^3) = \mathbf{9m}$ for some integer m, where $m = 3q^3$

ii) If $x = 3q + 1 \Rightarrow x^3 = (3q + 1)^3 = (3q)^3 + 3(3q)^2(1) + 3(3q)(1)^2 + (1)^3$

$[\because \text{since } (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= 27q^3 + 27q^2 + 9q + 1$

$= 9q(3q^2 + 3q + 1) + 1$

$= \mathbf{9m + 1}$ for some integer m,

where $m = q(3q^2 + 3q + 1)$

iii) If $x = 3q + 2 \Rightarrow x^3 = (3q + 2)^3$

$= (3q)^3 + 3(3q)^2(2) + 3(3q)(2)^2 + (2)^3$

$[\because \text{since } (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3]$

$= 27q^3 + 54q^2 + 36q + 8$

$= 9q(3q^2 + 6q + 4) + 8$

$= \mathbf{9m + 8}$ for some integer m,

where $m = q(3q^2 + 6q + 4)$

Hence, the cube of any positive integer is either of the form $9m$, $9m+1$ or $9m + 8$

OR

29. Let x be any positive integer and $b = 3$

$\therefore$ Applying Euclid's Division Algorithm

$x = 3q + r$ where $0 \leq r < 3$

$\therefore$ The possible remainders are 0, 1, 2

$\therefore$ $x = 3q$ or $3q + 1$ or $3q + 2$

Now,

i) If $x = 3q \Rightarrow x^2 = (3q)^2$

$= 9q^2$

$= 3(3q^2)$

$= \mathbf{3m}$ for some integer m, where $m = 3q^2$

ii) If $x = 3q + 1 \Rightarrow x^2 = (3q + 1)^2$

$= 9q^2 + 6q + 1$

$= 3q(3q + 2) + 1$

$= \mathbf{3m + 1}$ for some integer m, where $m = q(3q + 2)$

iii) If $x = 3q + 2 \Rightarrow x^2 = (3q + 2)^2$

$= 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2 + 4q + 1) + 1$

$= \mathbf{3m + 1}$ for some integer m,

where $m = 3q^2 + 4q + 1$

Hence, the square of any positive integer is either of the form 3m or $3m + 1$

30. In Δ ABC, we have

DE || BC

$\Rightarrow \angle ADE = \angle ABC$ and $\angle AED = \angle ACB$ [Corresponding angles]

Thus, in triangles ADE and ABC, we have

$\angle A = \angle A$ [Common]

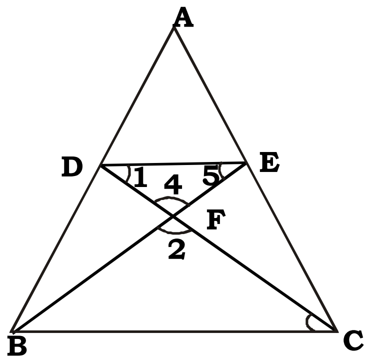
$\angle ADE = \angle ABC$

and, $\angle ADE = \angle ACB$

$\therefore$ $\Delta AED \sim \Delta ABC$

$\Rightarrow \dfrac{AD}{AB} = \dfrac{DE}{BC}$

We have, $\dfrac{AD}{AB} = \dfrac{5}{4}$



$\Rightarrow \quad \frac{DB}{AD} = \frac{4}{5}$

$\Rightarrow \quad \frac{DB}{AD} + 1 = \frac{4}{5} + 1$

$\Rightarrow \quad \frac{DB + AD}{AD} = \frac{9}{5}$

$\Rightarrow \quad \frac{AB}{AD} = \frac{9}{5} \Rightarrow \frac{AD}{AB} = \frac{5}{9}$

$\Rightarrow \quad \frac{DE}{BC} = \frac{5}{9}$

In ΔDFE and ΔCFG, we have

$\angle 1 = \angle 3$ [Alternate interior anges]

$\angle 2 = \angle 4$ [Vertically interior anges]

Therefore, by AA -similarity criterion, we have

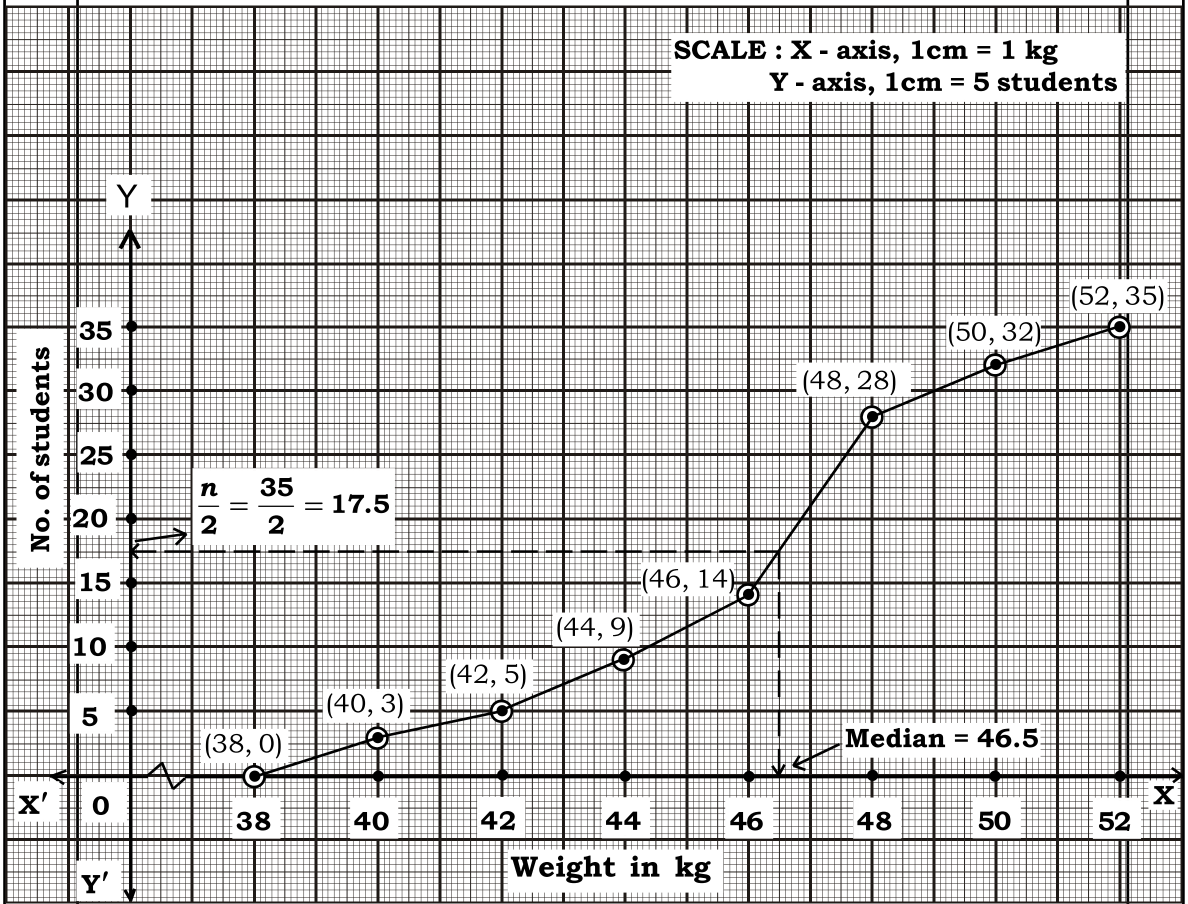
$\Delta DFE \sim \Delta CFG$

$\Rightarrow \quad \frac{Area(DFE)}{Area(CFB)} = \frac{DE^2}{BC^2}$

$\Rightarrow \quad \frac{Area(DFE)}{Area(CFB)} = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$ [Using (i)]

31.

Weight (in kg)	**Number of students**	**Weight (in kg)**	**C.f**	**Points to beplotted**
36 - 38	0	Less than 38	0	(38, 0)
38 - 40	3 – 0 = 3	Less than 40	3	(40, 3)
40 - 42	5 – 3 = 2	Less than 42	5	(42, 5)
42 - 44	9 – 5 = 4	Less than 44	9	(44, 9)
44 - 46	14 – 9 = 5	Less than 46	14 $\leftarrow$ *c.f.*	(46, 14)
46 - 48	28 – 14 = 14 $\leftarrow f$	Less than 48	28	(48, 28)
48 - 50	32 – 28 = 4	Less than 50	32	(50, 32)
50 - 52	35 – 32 = 3	Less than 52	35	(52, 35)



∴ Median from the graph is 46.5 kg

Now, = $\frac{n}{2} = \frac{35}{2}$ = 17.5 which lies in the class 46 - 48. (See the table)

∴ **Median class is 46 - 48.**
l = 46, h = 2, f = 14, c.f. = 14

$$\textbf{Median} = l + \left(\frac{\frac{n}{2} - c.f}{f}\right) \times h$$

$$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$$

$$= 46 + \frac{3.5}{7}$$

$$= \mathbf{46 + 0.5}$$

∴ Median = 46.5 kg.

Hence the median is same as obtained from the graph.

32. Let X and Y be two cars starting from points A and B respectively. Let the speed of car X and xkm/ hr and that of car Y be ykm/hr.

Case I When two cars move in the same directions.

Suppose tow cars meet at a point Q. Then,

Distance travelled by car X = AQ,

Distance travelled by carYX = BQ,

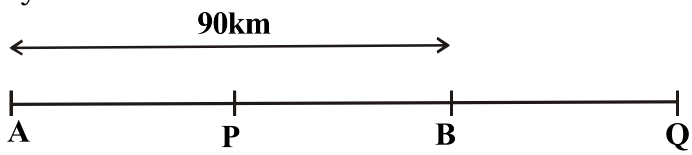
It is given that two cars meet in 9 hours.

$\therefore$ Distance travelled by car X in 9 hours = 9xkm.

$\Rightarrow$ AQ = 9x

Distance travelled by car y 0n 9 hours = 9ykm.

$\Rightarrow$ BQ = 9y



Clearly, AQ – BQ = AB [$\because$ AB =90km]

$\Rightarrow$ 9x – 9y = 90 ... (i)

$\Rightarrow$ x – y = 10

Case II When two cars move in opposite directions :

Suppose two cars meet at point P. Then,

Distance travelled by car X = AP,

Distance travelled by car Y = BP,

In this case, two cars meet in 9/7 hours.

$\therefore$ Distance travelled by car X in $\frac{9}{7}$ hours = $\frac{9}{7}$xkm

$\Rightarrow$ BP = $\frac{9}{7}y$

Clearly, AP + BP = AB

$\Rightarrow$ $\frac{9}{7}x + \frac{9}{7}y = 90$

$\Rightarrow$ $\frac{9}{7}(x + y) = 90$

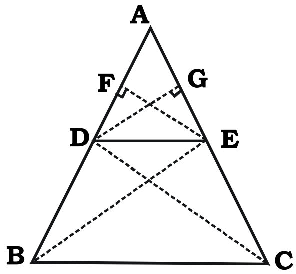
$\Rightarrow$ $(x + y) = 70$ ... (ii)

Solving equations (i) and (ii), we get

x = 40 and y = 30.

Hence, speeed of car X is 40km/hr and speed of car Y is 30km/hr.

33. Given : A Δ ABC, in which DE || BC such that DE intersects AB and AC at D and E respectively.



To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE and CD and draw EF ⊥ AB, DG ⊥ AC.

Proof : We have :

$$ar(\Delta ADE) = \frac{1}{2} \times AD \times EF \quad \text{.... [∵ Base = AD and height = EF]}$$

$$\text{and } ar(\Delta DBE) = \frac{1}{2} \times DB \times EF \quad \text{.... [∵ Base = DB and height = EF]}$$

$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF} = \frac{AD}{DB} \quad \text{.... (i)}$$

$$\text{Again, } ar(\Delta ADE) = \frac{1}{2} \times AE \times DG \quad \text{.... [∵ Base = AE and height = DG]}$$

$$\text{and } ar(\Delta DCE) = \frac{1}{2} \times EC \times DG \quad \text{.... [∵ Base = EC and height = DG]}$$

$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta DCE)} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC} \quad \text{.... (ii)}$$

Now, ΔDBE and ΔDCE are on the same base DE and between the same parallels DE and BC.

∴ ar (ΔDBE) = ar (ΔDCE)

Therefore, equation (ii) becomes,

$$\frac{ar(\Delta ADE)}{ar(\Delta DBE)} = \frac{AE}{EC} \quad \text{.... (iii)}$$

From (i) and (iii), we get : $\boxed{\mathbf{\frac{AD}{DB} = \frac{AE}{EC}}}$

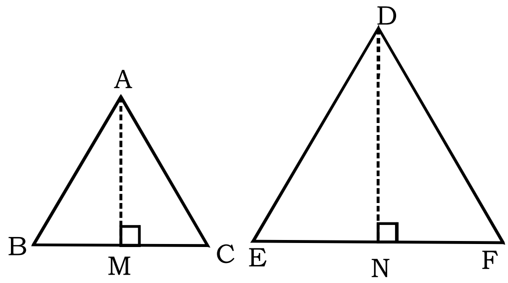
OR

33. **Given :** $\Delta ABC \sim \Delta DEF$

To prove :

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Construction : Draw $AM \perp BC$ and $DN \perp EF$



Proof : $\Delta ABC \sim \Delta DEF$

$\therefore \quad \angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ .... (i)

Now, $ar(\Delta ABC) = \left(\frac{1}{2} \times BC \times AM\right)$ and

$ar(\Delta DEF) = \left(\frac{1}{2} \times EF \times DN\right)$

$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\left(\frac{1}{2} \times BC \times AM\right)}{\left(\frac{1}{2} \times EF \times DN\right)} = \frac{BC}{EF} \times \frac{AM}{DN}$ .... (ii)

Now, in ΔAMB and ΔDNE we have :

$\angle AMB = \angle DNE$ .... (Each being a right angle)

and $\angle B = \angle E$ .... [Using (i)]

$\therefore \quad \Delta AMB \sim \Delta DNE$ .... (AA similarity)

and so, $\frac{AM}{DN} = \frac{AB}{DE}$ ... (corresponding sides of similar triangles)

$\therefore \quad \frac{AM}{DN} = \frac{BC}{EF}$ .... (iii) ..... [Using (i) we have $\frac{AB}{DE} = \frac{BC}{EF}$]

From (ii) and (iii) we get :

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{BC}{EF} \times \frac{BC}{EF}\right) = \frac{BC^2}{EF^2}$$

Similarly, we have :

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} \text{ and } \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AC^2}{DF^2}$$

Hence, $\boldsymbol{\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}}$

34. L.H.S. $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing Numerator & Denominator by Sin A

$= \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$

$= \dfrac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec}^2 A - \cot^2 A)}{(1 + \cot A - \operatorname{cosec} A)}$ .... $(\because \operatorname{cosec}^2 A - \cot^2 A = 1)$

$= \dfrac{(\cot A + \operatorname{cosec} A) - (\operatorname{cosec} A + \cot A)(\operatorname{cosec} A - \cot A)}{(1 + \cot A - \operatorname{cosec} A)}$

$= \dfrac{(\cot A + \operatorname{cosec} A)(1 - \operatorname{cosec} A + \cot A)}{(1 + \cot A - \operatorname{cosec} A)}$

$= \cot A + \operatorname{cosec} A$

= R.H.S.

$\therefore$ **L.H.S. = R.H.S. .... Hence proved.**

❖❖❖❖❖